0925 - DIS 202

Thursday, September 24, 2020 3:26 PM

List all the ways to represent a line.

- vector $V_1 + tV_2 \quad t \in [-\infty, \infty]$
- parametric
$$x = k_1 t + b_1$$
$$y = k_2 t + b_2$$
$$z = k_3 t + b_3$$
- Cartesian $\mathbb{R}^2$ $y = kx + b$
$$\mathbb{R}^3 \quad \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$
- Two planes intersect at a line
$$A_1 x + B_1 y + C_1 z = D_1$$
$$A_2 x + B_2 y + C_2 z = D_2$$
  1. solve two of x, y, z in terms of the other
  2. $\langle A_1, B_1, C_1 \rangle \times \langle A_2, B_2, C_2 \rangle$
     product of normals gives direction

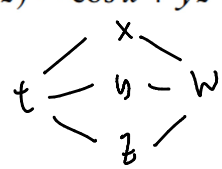
$Ax + By + Cz = d$ $\quad (x_0, y_0, z_0)$

- Use a plane & a point to specify a normal
  normal vec : $\langle A, B, C \rangle$
  $(x_0, y_0, z_0) + \langle A, B, C \rangle t$
- Two pts define a line
  p, q
  1. $p + t(q - p) \quad t \in [-\infty, \infty]$
  2. interpolating $tq + (1-t)p$
     $t \in [0,1]$ - line segment
     $t \in [-\infty, \infty]$ - line

$1 \to 1 \qquad 3 \to 1 \qquad 1 \to 3$

11. Find $\frac{\partial w}{\partial t}$ for $w(t) = \varphi(r(t))$, $\varphi(x, y, z) = \cos x + yz^2$, $r(t) = t\mathbf{i} + \ln(t^2)\mathbf{j} + e^t\mathbf{k}$.

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$
$$= \vec{\nabla}\varphi \cdot r'(t)$$



$x = t \quad y = \ln(t^2) \quad z = e^t$

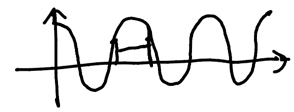
List all the ways to represent a plane.

- $Ax + By + Cz = D$
- $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$
- $P = (x_0, y_0, z_0)$
  normal vec $(u, v, w) = \vec{n}$
  $u(x - x_0) + v(y - y_0) + w(z - z_0) = 0$
  $\{P + \vec{x} \mid \vec{x} \cdot \vec{n} = 0\}$
- 3 pts $P \; Q \; R$
  $t(Q - P) + s(R - P) + P \quad t, s \in \mathbb{R}$
  $P = (a_1, a_2, a_3)$
  $Q = (b_1, b_2, b_3)$
  $R = (c_1, c_2, c_3)$
  $\overrightarrow{PQ} \times \overrightarrow{PR} = (A, B, C)$
  $A(x - a_1) + B(y - a_2) + C(z - a_3) = 0$

Given $r = f(\theta)$, find the shortest interval that it takes to repeat.

$r = 3\cos\left(\frac{5}{6}\theta\right) + 1$

$\theta_1 \quad \theta_2$ same pt (& $\theta_1 + t$, $\theta_2 + t$ same pt)



$\theta_1 = \theta_2 + k\pi \qquad \cancel{3}\cos\left(\frac{5}{6}(\theta_2 + k\pi)\right) + 1/3 = \pm\left(\cancel{3}\cos\left(\frac{5}{6}\theta_2\right) + 1/3\right)$