3. (10 points) Compute the following limit or show it does not exist:

$x,y,z \to (0,0,0)$

$$\lim_{(x,y)\to(0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$$

$z = 0$ $\frac{xy}{x^2+y^2}$ $x = 0$ $y \to 0$ 0

$x = y$ $\to 0$ $\frac{1}{2}$

$x = y, z = 0$

---

① $|x|, |y| \le r$

② $1+x^2 \ge 1$ and $1+x^2 \ge x^2$

$\frac{1}{\sqrt{x^4+1}} \le \frac{1}{x^2}$

$2xy \le x^2+y^2 = r^2$

$x = r\cos\theta$, $|\cos\theta| \le 1 \Rightarrow |x| = r$

$y = r\sin\theta$ $|\sin\theta| \le$

5. Use polar coordinates to find the limit. at (x,y) distance from $(0,0)$ is r

then $|x| \le r$ $|y| \le r$

$x^2+y^2 = r^2$

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$$

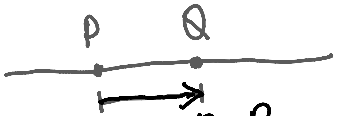
$$-2r \le -\left|\frac{x^3+y^3}{x^2+y^2}\right| \le \frac{x^3+y^3}{x^2+y^2} \le \left|\frac{x^3+y^3}{x^2+y^2}\right| = \frac{|x^3+y^3|}{|x^2+y^2|} \le \frac{|x^3|+|y^3|}{r^2} \le \frac{r^3+r^3}{r^2} = 2r$$

$(x,y) \to 0$ $r \to 0$

0927 - DIS 202

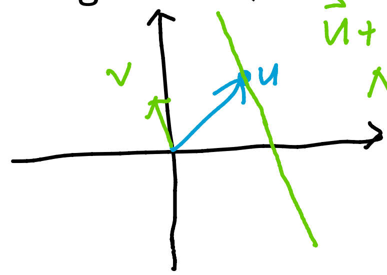
Friday, September 25, 2020 12:49 AM

A word about parametrizing the line segment PQ:



P

$p + (Q-P)t \quad t \in [0,1]$



$\vec{u} + \vec{v}t$

Note: if $\vec{u} \neq 0$ then line does not pass through $\vec{v}$

T/F from Steel SP2012 Midterm 1:

T 1. For any vectors $\vec{a}$ and $\vec{b}$, if $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$, then $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$.

F 2. If $\vec{u}(t)$ and $\vec{v}(t)$ are differentiable vector-valued functions, then $\frac{d}{dt}[\vec{u}(t) \times \vec{v}(t)] = \vec{u}'(t) \times \vec{v}'(t)$.

$(a_1 b_2 - b_1 a_2 \cdots) \quad (a_1' b_2 + a_1 b_2') - (b_1' a_2 + b_1 a_2')$

$= u'(t) \times v(t) + u(t) \times v'(t)$

$u \quad (a_1(t), a_2(t), a_3(t))$

$v \quad (b_1(t), b_2(t), b_3(t))$

F 3. If $f(x,y) \to L$ as $(x,y) \to (a,b)$ along every straight line, then $\lim_{(x,y)\to(a,b)} f(x,y) = L$.

14.2.44

$y > x^4 \quad f(x,y) \to 0$

$y \leq 0 \quad (x,y) \to 0$ along straight lines

T 4. If all partials of F exist and are continuous everywhere, and $\nabla F(a,b,c) \neq \vec{0}$, then the equation $F(x,y,z) = F(a,b,c)$ defines a surface near (a,b,c).

$\vec{\nabla} F \cdot \vec{u} = 0 \quad \vec{\nabla} F \neq 0 \quad \{\vec{u} \mid \vec{\nabla} F \cdot \vec{u} = 0\} \approx \mathbb{R}^2$ locally

F 5. Let $R(x,y) = f(x,y) - (f(a,b) + \underbrace{f_x(a,b)}_{\text{constant}}\underbrace{(x-a)}_{\to 0} + \underbrace{f_y(a,b)}_{\text{constant}}\underbrace{(y-b)}_{\to 0})$. Then f is differentiable at (a,b) if and only if $R(x,y) \to 0$ as $(x,y) \to (a,b)$.

continuous $\quad R(x,y) \to 0 \implies f(x,y) - f(a,b) \to 0$

F 6. If $f_x(a,b)$ and $f_y(a,b)$ exist, then f is differentiable at (a,b).

T 7. If f is differentiable at (a,b), and $\nabla f(a,b) \neq \vec{0}$, then the derivative of f at (a,b) in the direction tangent to the curve $f(x,y) = f(a,b)$ is zero.

u

level curve $\quad \vec{\nabla} f \cdot \hat{u} = 0 = D_{\hat{u}} f$

T 8. If all second partial derivatives of f exist and are continuous everywhere, then $f_{xy} = f_{yx}$.

Clairaut

Evans FA2010 Midterm 1: