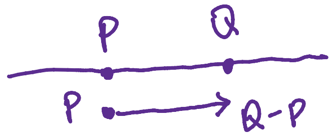
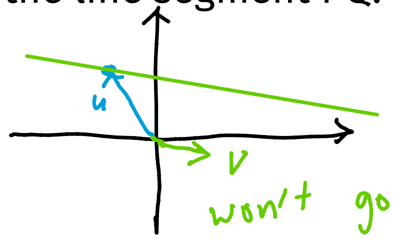
0927 - DIS 204

Sunday, September 27, 2020 9:22 PM

A word about parametrizing the line segment PQ:



$P+(Q-P)t \quad t\in[0,1]$



won't go through v unless $\vec{u}=0$

T/F from Steel SP2012 Midterm 1:

$a,b\neq 0$ $\quad\perp\quad$ $//$

T 1. For any vectors $\vec{a}$ and $\vec{b}$, if $\vec{a}\cdot\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{0}$, then $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$.

F 2. If $\vec{u}(t)$ and $\vec{v}(t)$ are differentiable vector-valued functions, then $\frac{d}{dt}[\vec{u}(t)\times\vec{v}(t)]=\vec{u}'(t)\times\vec{v}'(t)$.

$u\ (a_1(t), a_2(t), a_3(t))$ $\quad u\times v\ (a_2b_3-a_3b_2, \cdots \quad \cdots)$

$(u\times v)' = u'\times v + u\times v'$ $\quad v\ (b_1(t), b_2(t), b_3(t))$ $\quad (a_2'b_3 + a_2b_3' - a_3'b_2 - a_3b_2', \cdots)$

F 3. If $f(x,y)\to L$ as $(x,y)\to(a,b)$ along every straight line, then $\lim_{(x,y)\to(a,b)} f(x,y) = L$.

14.2.44

T 4. If all partials of F exist and are continuous everywhere, and $\nabla F(a,b,c)\neq\vec{0}$, then the equation $F(x,y,z)=F(a,b,c)$ defines a surface near (a,b,c).

$\{\vec{u}\mid\vec{\nabla}F\cdot\vec{u}=0\}\approx\mathbb{R}^2$ locally. Level surface: locally similar to the set of directions where $D_uf=0$

F 5. Let $R(x,y)=f(x,y)-(f(a,b)+f_x(a,b)\underbrace{(x-a)}_{\to 0}+f_y(a,b)\underbrace{(y-b)}_{\to 0})$. Then f is differentiable at (a,b) if and only if $R(x,y)\to 0$ as $(x,y)\to(a,b)$.

$f_x(a,b),\ f_y(a,b)$ constants $\quad R(x,y)\to 0:\ f(x,y)-f(a,b)\to 0$

$R(x,y)=o(x^2+y^2)$, or $\lim_{(x,y)\to(a,b)}\frac{R(x,y)}{|x|+|y|}\to 0$

F 6. If $f_x(a,b)$ and $f_y(a,b)$ exist, then f is differentiable at (a,b).

T 7. If f is differentiable at (a,b), and $\nabla f(a,b)\neq\vec{0}$, then the derivative of f at (a,b) in the direction tangent to the curve $\underbrace{f(x,y)=f(a,b)}_{\text{level set}}$ is zero. $\vec{\nabla}f\cdot\vec{u}=0$

$D_{\vec{u}}f=\vec{\nabla}f\cdot\vec{u}$ ($\vec{u}$ is unit vector) $\quad D_{\vec{u}}f=0$

T 8. If all second partial derivatives of f exist and are continuous everywhere, then $f_{xy}=f_{yx}$.

Wagoner FA2000 Midterm 1:

Problem #1 From the diagrams below select the picture which best represents each of the following parametrized curves.

(A)-(4) $a(t) = (cos(t) - 1, sin(t) - t)$ (B)-(5) $b(t) = (3cos(t) + 1, sin(t) - 2)$

(C)-(3) $c(t) = (t(t-1), t(t-1)(t-2))$ (D)-(7) $d(t) = (cos(3t), sin(2t))$

(E)-(8) $c(t) = (e^t cos(t), e^t sin(t))$ (F)-(1) $f(t) = (t, t^2 + sin(t))$