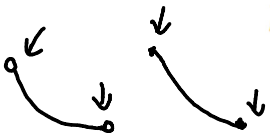
1007 - DIS 202

Tuesday, October 6, 2020 11:11 PM

1. Find the maximum and minimum values of

$$f(x, y) = 3x^2 + 6y^2 \text{ on the disk } x^2 + y^2 \le 4.$$



continuous bounded closed set

must achieve the supremum & infimum

- $x^2+y^2 < 4$ $f_x = f_y = 0$, check each pt

$f_x = 6x$ $f_y = 12y$ $\to x = y = 0$ $D(0,0) = f_{xx} f_{yy} - f_{xy}^2 = 6 \cdot 12 - 0^2 > 0$

$f_{xx} > 0 \to$ minimum

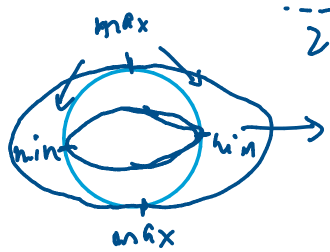
- $x^2+y^2 = 4 = g$ Lagrange Multiplier

$\nabla f = \langle 6x, 12y \rangle$ $\nabla g = \langle 2x, 2y \rangle$

$6x = \lambda 2x$ if $x, y \neq 0$ λ ? $4 \neq 2$

$12y = \lambda 2y$ $(0, \pm 2)$ $(\pm 2, 0)$

$3x^2 + 6y^2 = a$ 24 12



larger

$(0, \pm 2)$ global max

$(\pm 2, 0)$ neither max/min

$(0,0)$ global min

2. Find the extreme values of f on the region described by the inequality.

$$f(x, y) = x^2 - 3xy + y^2 - 4x, \qquad x^2 \le y.$$

$\leftarrow$ $x^2 < y$

$x^2 = y$

$f_x = 2x - 3y - 4$ $f_x = f_y = 0$ $3x = 2y$

$f_y = -3x + 2y$ $x = \frac{2}{3} y$

$2 \cdot \frac{2}{3} y - 3y - 4 = -\frac{5}{3} y - 4 = 0$

$x = \frac{-8}{5}$ $y = \frac{-12}{5} < 0 < x^2$

$\to$ $x^2 = y$ case Lagrange Multiplier : $g(x,y) = y - x^2 = 0$

$\begin{pmatrix} 2x - 3y - 4 \\ -3x + 2y \end{pmatrix} = \lambda \begin{pmatrix} -2x \\ 1 \end{pmatrix}$ $2x - 3y - 4 = -2\lambda x$

$2x(-3x + 2y) = \lambda 2x$

Extreme values at its solution

$4x^3 - 9x^2 + 2x - 4 = 0$ minimum $\quad -6x^2 + 4xy + 2x - 3y - 4 = 0$

$x \approx 2.271 \quad -6x^2 + 4x^3 + 2x - 3x^2 - 4 = 0$

3. (Hutchings Fa03-2) Find the maximum and minimum value of the function

$$f(x, y) = (x-1)^2 + (y-1)^2 \text{ on the disk } x^2 + y^2 \leq 1.$$

$f_x = 2(x-1) \quad f_y = 2(y-1) \quad f_x = f_y = 0$ at $(1,1)$

$1^2 + 1^2 > 1$

$g = x^2 + y^2 = 1$

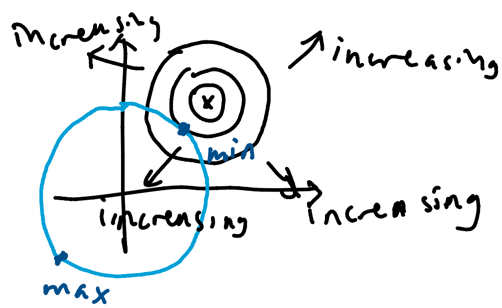
$2(x-1) = \lambda 2x$

$2(y-1) = \lambda 2y$

$-2 = 2(\lambda - 1)x = 2(\lambda-1)y$

$x = y$

$\pm\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$



$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ global min

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ global max

4. (Givental Fa15-2) Find the maximum and minimum values of the function

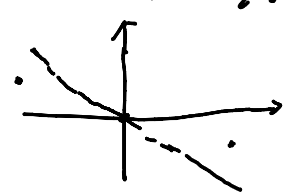
$2(x^2 + 3xy + \frac{9}{2}y^2) = 2\left((x + \frac{3}{2}y)^2 + \frac{9}{4}y^2\right)$

$$f(x, y) = x \text{ in the region } 2x^2 + 6xy + 9y^2 \leq 9.$$

$x = -\frac{3}{2}y$

graph: tilted plane $\quad$ no critical pt

also, $f_x = 1 \neq 0$



Boundary: Lagrange Multiplier

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 4x + 6y \\ 6x + 18y \end{pmatrix} \quad \lambda \neq 0$ b.c. $1 = \lambda(4x + 6y)$

$\rightarrow 6x + 18y = 0$

$3y = -x \leftarrow$

$2x^2 + 6xy + 9y^2$ $\quad \cancel{1 = \lambda 2x}$

$= \cancel{18y}^2 + 6\cancel{(-3y)}y + 9y^2 = 9$

$y = \pm 1$

$(3, -1)$ max on boundary

$(-3, 1)$ min on boundary

$(3, -1)$ is a global max $\quad (-3, 1)$ is global min (region is ellipse)

5. Evaluate

$$\int_0^2 \boxed{\int_0^3 x^2 e^y \, dx} \, dy.$$

constant for fixed y

$$\int_0^2 \left[\frac{x^3}{3} e^y \right]_0^3 dy$$