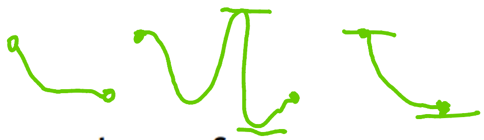
1007 - DIS 204

Tuesday, October 6, 2020 11:11 PM



1. Find the maximum and minimum values of

continuous — bounded, close set

$$f(x, y) = 3x^2 + 6y^2 \text{ on the disk } x^2 + y^2 \le 4.$$

$-\ x^2+y^2<4$ $\quad f_{xx}=6>0$

$f_x = f_y = 0$ $\quad D(0,0)=6\cdot 12-0^2 = 72>0$

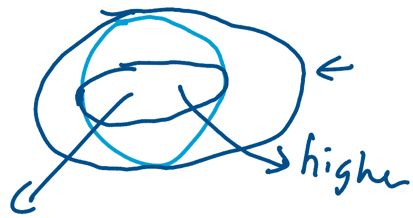
$6x = 12y = 0$

$(0,0)$ min

$3x^2+6y^2=a$

$x^2+2y^2=\frac{a}{3}$

$\left(\frac{x}{\sqrt{2}}\right)^2+y^2=\frac{a}{6}$



$-\ x^2+y^2=4$

$\begin{pmatrix}6x\\12y\end{pmatrix}=\lambda\begin{pmatrix}2x\\2y\end{pmatrix}$ $\quad x, y\ne 0 \to \lambda ?\ 4\ne 2$

$(0,\pm 2)$ $\quad (\pm 2, 0)$ $\quad$ max, min on boundary

$(\pm 2,0)$ neither max nor min

$(0,0)$ global min $\quad (0,\pm 2)$ global max

3. (Hutchings Fa03-2) Find the maximum and minimum value of the function

$$f(x, y) = (x-1)^2 + (y-1)^2 \text{ on the disk } x^2 + y^2 \le 1.$$

$f_x = 2(x-1)$ $\quad f_x = f_y = 0$ $\quad x=y=1$ $\quad 1^2+1^2>1$

$f_y = 2(y-1)$

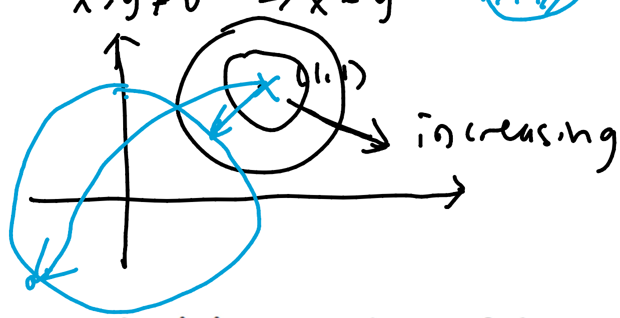
$g(x,y) = x^2+y^2=1$

$\begin{pmatrix}2(x-1)\\2(y-1)\end{pmatrix}=\lambda\begin{pmatrix}2x\\2y\end{pmatrix}$ $\quad -2 = 2(\lambda-1)x = 2(\lambda-1)y$

$x, y\ne 0 \to x=y$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ global min

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ global max



4. (Givental Fa15-2) Find the maximum and minimum values of the function

$$f(x, y) = x \text{ in the region } 2x^2 + 6xy + 9y^2 \le 9.$$

tilted plane $\qquad$ no critical pt

$\rightarrow \quad 2x^2+6xy+9y^2=9 \quad \cdots \; 9$

$$\begin{pmatrix}1\\0\end{pmatrix} = \lambda\begin{pmatrix}4x+6y\\6x+18y\end{pmatrix}$$

$\leftarrow \lambda \neq 0$

$\leftarrow 6x+18y=0$

$x=-3y$

$\cancel{18y^2}-\cancel{18y^2}+9y^2=9 \quad y=\pm 1$

$(3,-1)$ global max $\qquad (-3,1)$ global min

5. Evaluate

partial : assuming all other var const.

$$\int_0^2\int_0^3 x^2e^y\,dx\,dy.$$

when finished with integrating $\int_a^b g(x)\,dx$, x disappear in the result

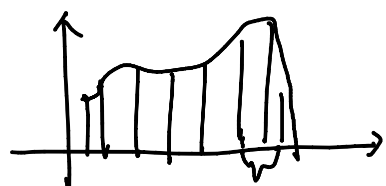
$$\int_0^3 x^2e^y\,dx = \left[\frac{x^3}{3}e^y\right]_0^3 = 9e^y-0 = 9e^y$$

$$\int_0^2 f(y)\,dy$$

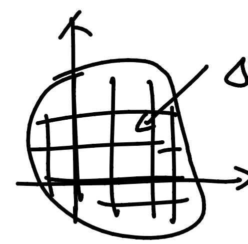
$$\downarrow$$

$$\int_0^2 9e^y\,dy = \left[9e^y\right]_0^2 = \boxed{9e^2-9}$$

$$\int_0^3 x^2e^y\,dx = e^y\int_0^3 x^2\,dx$$



Δx



ΔA

6. Evaluate

$$\iint_R xy^2\sqrt{x^2+y^3}\,dA \qquad R=[0,3]\times[0,2].$$

$$=\int_0^3\int_0^2 xy^2\sqrt{x^2+y^3}\,dy\,dx$$

$u=x^2+y^3$

$du=3y^2\,dy$

$$=\int_0^3\int_{x^2}^{x^2+8}\frac{x}{3}(\cancel{3y^2})\sqrt{u}\,\frac{du}{\cancel{3y^2}}\,dx$$

$$=\int_0^3\int_{x^2}^{x^2+8}\frac{x}{3}\sqrt{u}\,du\,dx$$

$$\int_m^N ax^2+bx+c\,dx$$

$$= \int_0^3 \frac{x}{3} \left[\frac{2}{3} u^{\frac{3}{2}} \right]_{x^2}^{x^2+8} dx$$

$$= \int_0^3 \frac{x}{3} \left(\frac{2}{3}(x^2+8)^{\frac{3}{2}} - \frac{2}{3} x^3 \right) dx$$

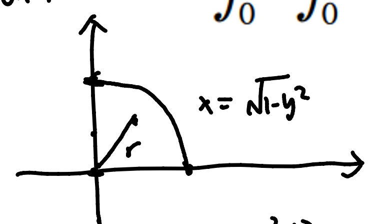
$$= \int_0^3 \frac{2x}{9} (x^2+8)^{\frac{3}{2}} \, dx - \int_0^3 \frac{2}{9} x^4 \, dx$$

$v = x^2+8, \; dv = 2x\,dx$

7. (Hutchings Fa03-2) Calculate

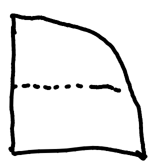
$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)^{2020} dx\,dy$$

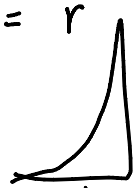
$\iint_R \cdots dA$



$x^2+y^2 = r^2$

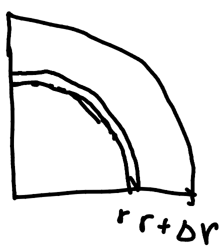
$(x^2+y^2)^{2020} = r^{4040}$





volume under surface f

$f(x,y) = (x^2+y^2)^{2020}$



$\frac{\pi r}{2} \Delta r \cdot r^{4040} \to \int_0^1 \frac{\pi r}{2} r^{4040} dr$

8. (Hutchings Fa03-2) Calculate

"dxdy"

$$\int_0^1 \int_{x^{2/3}}^1 x \cos(y^4) dy\,dx.$$

y^3

① x boundary : constants

$\boxed{0 \le x} \le 1$

y boundary : depend on x

$x^{2/3} \le y \le 1$

$0 \le x^2 \le y^3 \le 1$

$x\cos(y^4)$ continuous

along x, $\forall y$ fixed

y, $\forall x$ fixed

$0 \leq x^2 \leq y^3 \leq 1$

$\begin{cases} 0 \leq y \leq 1 \\ 0 \leq x^2 \leq y^3 \end{cases}$ $\int_0^1 \int_0^{y^{3/2}} x \cos(y^4)\, dx\, dy$

(2) y boundary absolute/constants

x boundary dep on y

$0 \leq y \leq 1$

$x^{2/3} \leq y$

$0 \leq x \leq y^{3/2}$

$\cos(y^4) \int_0^{y^{3/2}} x\, dx = \cos(y^4) \cdot \left[\frac{x^2}{2}\right]_0^{y^{3/2}}$

$= \cos(y^4) \cdot (y^3 - 0)$

$\int_0^1 y^3 \cos(y^4)\, dy \qquad u = y^4 \qquad du = 4y^3\, dy$