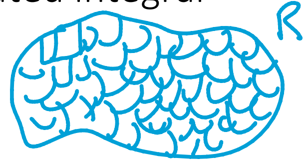
1009 - DIS 204

Thursday, October 8, 2020 8:29 PM

- Multiple integral vs Iterated integral

Double $\iint_R \cdots \, dA$
(region on $\mathbb{R}^2$)

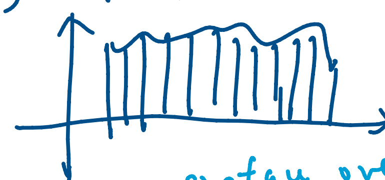


pieces → smaller
number the pieces
take representative
$(x_i, y_i) \in A_i$
$\sum_{i=1}^{n} f(x_i, y_i)\, \Delta A_i$
$n \to \infty$

in most practical situations here:
convert to "iterated integral"

$$\int_a^b \boxed{\int_c^d f(x,y)\,dy}\, dx$$

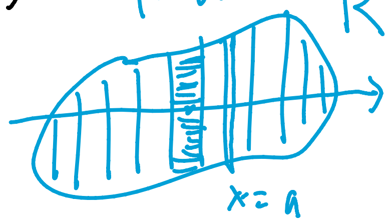
$f(a,y)$ $x = a$ const.



$$\int_a^b \boxed{\int_c^d xy^2\, dy}\, dx$$

$$\left[\frac{x}{3} y^3\right]_c^d = \frac{x}{3}(d^3 - c^3)$$

$f(x,y)$ surface over R



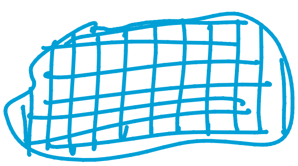
"dy dx"

$$x \mapsto \int_c^d f(x,y)\, dy$$

$$x \mapsto x \cdot \frac{d^3 - c^3}{3}$$

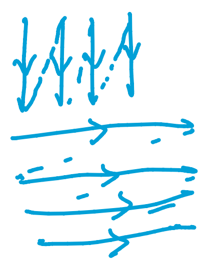
① Multiple integral : arbitrary shape
Iterated integral : rectangles

② "Conditionally convergent series"



$\iint dy\, dx$

$\iint dx\, dy$



"rearranging infinite series"
not obvious that
$\underbrace{\iint dx\,dy \ ? \iint dy\,dx}_{? \ \iint_R dA}$

$f(x,y)$ continuous on $\mathbb{R}^2$ $\xrightarrow{\ ?\ \text{become}\ =\ }$

$$g(x,y) = \frac{xy}{x^2+y^2}$$
$$g(0,0) = 0$$

$$\int_{-1}^{1} \boxed{\int_{-1}^{1} \frac{xy}{x^2+y^2}\, dx}\, dy$$

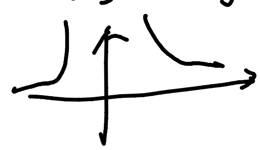
$g(x,y) = g(-x,y)$
when $y \neq 0$: inside $= 0$
$y = 0$: inside $= 0$

$g(x,y)$ not continuous in $[-1,1]\times[-1,1]$,
but iterated integral exists

$$f(x,y)=\frac{xy}{(x^2+y^2)^2} \qquad f(x,y) = f(-x,y) = f(x,-y)$$

along $x=y$, $f(x,y)\uparrow\infty$ as $x=y\to 0$

$$\frac{x^2}{(x^2+x^2)^2} = \frac{1}{4x^2}$$



as if you are matching $\frac{1}{x}$ & $\frac{1}{-x}$

$$\int_{-1}^{1}\int_{-1}^{1} f(x,y)\,dx\,dy = \int_{-1}^{1} 0\,dy = \int_{-1}^{1} 0\,dx = \int_{-1}^{1}\int_{-1}^{1} f(x,y)\,dy\,dx$$

$\iint_R f(x,y)\,dA = \text{DNE}$ in SV calc $\int_{-1}^{1}\frac{1}{x}\,dx = \text{DNE}$

$R=[-1,1]\times[-1,1]$

if you have something allowing matching $y(x)+y(-x)=0$ before integrating

Iterated integral exist $\to$ actually calculating the layers

$\iint_R \frac{xy}{(x^2+y^2)^2}\,dA$ DNE

- Jacobian matrix and u-substitution in several variables

$u=x^2+y^2$ $\quad du = 2y\,dy$
$\uparrow$
1 thing

$$\boxed{\begin{aligned} r\cos\theta &= x \\ r\sin\theta &= y\end{aligned}} \qquad \frac{\partial y}{\partial r} \qquad \frac{\partial x}{\partial \theta}$$

$\cancel{x = r\cos\theta + r^2}$
$\cancel{y = r\sin\theta + \theta}$

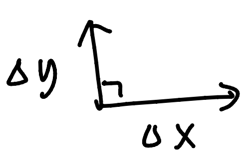
3. polar coordinates: $dxdy \to drd\theta$

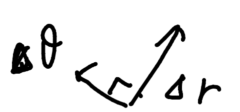
$dxdy = r\,drd\theta$

$$\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta}\end{bmatrix} \quad \text{Jacobian}$$

$dx\,dy = \det[\text{Jacobian}]\,drd\theta$

$$\begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix} = r(\cos^2\theta+\sin^2\theta) = r$$





$\Delta x = a\Delta r + b\Delta\theta$
$\Delta y = c\Delta r + d\Delta\theta$

$ad-bc$
$x_r y_\theta - x_\theta y_r$

5. the "volume of parallelotope" interpretation

$$x = 4u + w + v$$

$$y = u + w$$

$$z = w + 2v$$

One thing you learned + one thing still confusing. Thank you!