1012 - DIS 202

Sunday, October 11, 2020 11:50 PM

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$$X_1 = f_1(u_1, u_2, \cdots, u_n)$$
$$X_2 = f_2(u_1, u_2, \cdots, u_n)$$
$$X_3 = f_3(u_1, u_2, \cdots, u_n)$$
$$X_4 = f_4(u_1, u_2, \cdots, u_n)$$

4 variables

$$\cdot\ X_1 = r\cos u_1$$
$$\cdot\ X_2 = r\sin u_1 + r u_2 + r u_3$$
$$\cdot\ X_3 = u_2 + u_3 + u_4^2$$
$$\cdot\ X_4 = u_2 u_3 + u_1 u_4$$

r, u_1, u_2, u_3, u_4

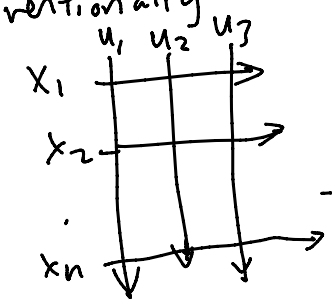
5 variables

variable # does not equal

→ not using it for u-sub.

$$\begin{bmatrix} \frac{\partial x_1}{\partial r} & \frac{\partial x_1}{\partial u_1} & \cdots & \frac{\partial x_1}{\partial u_4} \\ \frac{\partial x_2}{\partial r} & \frac{\partial x_2}{\partial u_1} & \cdots & \frac{\partial x_2}{\partial u_4} \\ \vdots & & \ddots & \vdots \\ \frac{\partial x_4}{\partial r} & \frac{\partial x_4}{\partial u_1} & \cdots & \frac{\partial x_4}{\partial u_4} \end{bmatrix}$$

Conventionally



4 rows,

5 cols

→ Jacobian matrix may not be square

When Jacobian matrix is $n \times n$,

det [Jacobian] is also called "Jacobian"

$$x = r\cos\varphi$$
$$y = r\sin\varphi$$

$$\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{bmatrix},$$

$$\det[J] = r(\cos^2\varphi + \sin^2\varphi) = r$$

$$F(r, u_1, \cdots, u_4) = (f_1(r, u_1, \cdots, u_4), f_2(r, u_1, \cdots, u_4), \cdots f_4(r, u_1, \cdots u_4))$$

r, u_1, u_2, u_3, u_4 ⋯ X_1 X_2 X_3 X_4

$\mathbb{R}^5 \to \mathbb{R}^4$

$\mathbb{R}^5 \to \mathbb{R}^5$

$\mathbb{R}^5 \to \mathbb{R}^4$

↳ 4 function, each $\mathbb{R}^5 \to \mathbb{R}$

$f_1(r, u_1, u_2, u_3, u_4) = x_1$, etc

Jacobian of f_1 is $f(r, u_1, u_2, u_3, u_4) \mapsto x$

$\left[\frac{\partial f_1}{\partial r} \quad \frac{\partial f_1}{\partial u_1} \quad \frac{\partial f_1}{\partial u_2} \quad \frac{\partial f_1}{\partial u_3} \quad \frac{\partial f_1}{\partial u_4}\right] \to \nabla f_1 \quad \mathbb{R}^5 \to \mathbb{R}$ (row)

$x = r \sin\varphi \sin\theta$ $\quad \nabla x = \langle \sin\varphi \sin\theta, r\cos\varphi \sin\theta, r \sin\varphi \cos\theta \rangle$

$y = r \sin\varphi \cos\theta$ $\quad$ also first row of $\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \varphi} & \frac{\partial z}{\partial \theta} \end{bmatrix}$

$z = r\cos\varphi$

$\mathbb{R} \to \mathbb{R}^n$

$f(t) = \langle t^2, t^3, \sin t \rangle$ (Jacobian matrix: columns)

Jacobian of f?

$f_1(t) = t^2$ $\quad f(t) = (f_1, f_2, f_3)$

$f_2(t) = t^3$

$f_3(t) = \sin t$

$$\begin{bmatrix} \frac{\partial f_1}{\partial t} \\ \frac{\partial f_2}{\partial t} \\ \frac{\partial f_3}{\partial t} \end{bmatrix} = \begin{bmatrix} 2t \\ 3t^2 \\ \cos t \end{bmatrix}$$

"Hessian" $f(x_1, x_2, \dots, x_n) \in \mathbb{R}$

$f: \mathbb{R}^n \to \mathbb{R}$ $\quad n \times n$ matrix

Jacobian $f: \mathbb{R}^n \to \mathbb{R}^n$ $\quad n \times n$ matrix