Math 53 DIS 202/204	**Jiazhen Tan (tanjz@berkeley.edu)**
Multivariable Calculus	*September 9, 2020*

Today's discussion. We will look at intersections of surfaces, look at some multi-input functions, then work on other multi-input and multi-output functions in breakout rooms.

Examples.

Rotated Curves+Generalizations.

1. Consider the curve $z = x^2$, $y = 0$ on the xz-plane, and generate a surface by rotating it around the z-axis. Which function has this surface as its graph?
2. Consider the curve $z = \frac{1}{x}$, $y = 0$ on the xz-plane, and generate a surface by rotating it around the z-axis. Which function has this surface as its graph?

Other higher dimensional shapes.

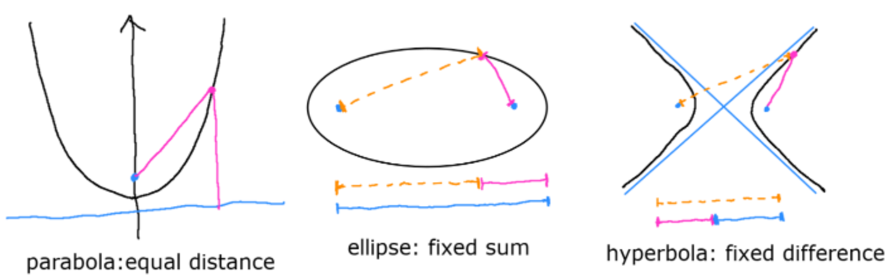
3. What does the shape $f(x, y) = x^2 - y^2$ look like?
4. What are the level sets of $f(x, y, z) = x^2 + y^2 + z^2$?
5. Find a vector function of the curve made by intersecting $x^2 + y^2 = 9$ and $x + z = 5$.
6. How would you parametrize a helix wrapping around a torus?

Consider $f(x, y) = 2\sqrt{x^2 + y^2}$.

7. What is the level set $f(x, y) = a$?

Consider the surface defined by the graph of f, which is the set of all points with coordinate $(x, y, f(x, y))$.

8. Find a vector function that represents the intersection of this surface and (a) the plane $x = 1$, (b) the plane $z = -2x$, (c) the plane $z = 2x$, (d) the plane $z = -2x + 1$.
9. What is the intersection of this surface and the paraboloid $z = 4x^2 + y^2$?



► **Exercise 10.5.1** tricks in finding the vertex+directrix of a parabola.

Problems. Those with an asterisk are harder.

1. **Exercise 12.6.21-28.**
2. For $\mathbf{r} = \langle t\cos(t), e^t, t^2\sin(2t)\rangle$, what is $\mathbf{r}'(1)$? What is the unit tangent vector at $t = 1$?

3.* Consider a circle centered at $(R, 0, 0)$ on the xz-plane, with radius r, and assume $r < R$. Make a donut/torus by rotating the circle around the z-axis. Can you parametrize this surface by a map

$$\mathbf{F} : (u, v) \mapsto \langle f(u, v), g(u, v), h(u, v)\rangle?$$

4. Let $f(x, y) = \sqrt{9 - x^2 - 4y}$.

   Evaluate $f(1, 1)$.

   What is the domain of the function?

5. Let $f(x, y, z) = \ln(9 - x^2 - 4y - z^2)$.

   Evaluate $f(1, 0, 1)$.

   What is the domain of the function?

6.* What is the shape of $f(x, y) = \dfrac{x^3 y}{x^2 + y^2}$ around the origin?

7. Draw a contour map for the functions by sketching the level curves: $f(x, y) = -2, -1, 0, 1, 2$ (you can use a graphing calculator if it is hard to simplify).
   (a) $f(x, y) = 4x^2 + y^2$.
   (b) $f(x, y) = xy + y^3$.
   (c) $f(x, y) = x^3 - 4x + xy^2$.
   (d) $f(x, y) = x^3 - 3xy^2$. [Look up “monkey saddle”.]
   (e) $f(x, y) = \sqrt{x} + \sqrt{y}$.
   (f) $f(x, y) = x^3 + 3x^2 y + 4y^3 + 60x$.

8.* Find a function $f(x, y)$ with level curves that looks like the following: (the idea is that as a changes, $f(x, y) = a$ goes from a closed curve to a curve extending to infinity)

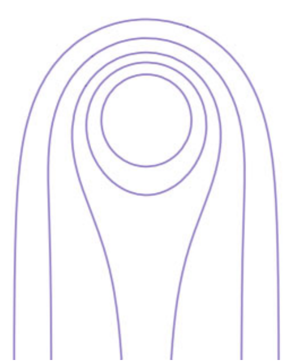


Figure 1: `https://www.desmos.com/calculator/7aagstvwwf`