Examples.

1. When does the line tangent to $f(x) = (x+3, x^2 - 2x + 1, x^3 - 2x^2 - x - 6)$ pass through the origin?

2. If both f_x, f_y exists at the origin, does that mean there is a tangent plane at the origin?

   `https://mathinsight.org/differentiability_multivariable_subtleties`

3. If both f_x, f_y exists at the origin, does that mean it is continuous at the origin?

   Reconsider the example
   $$f(x,y) = \frac{xy}{x^2 + y^2}.$$
   Write it in polar coordinates
   $$g(r,\theta) = \frac{r\cos\theta r\sin\theta}{r^2} = \frac{1}{2}\sin(2\theta).$$

4. For the functions below, can their tangent plane be horizontal?
   (a) $f(x,y) = 4x^2 + y^2$.
   (b) $f(x,y) = xy + y^3$.
   (c) $f(x,y) = \sqrt{x} + \sqrt{y}$.

5. In single variable calculus, how do you know if $f(x) = x^3 - 3x$ has a local maximum, minimum or a stationary point?

   How do you define a local maximum for single variable functions?

   Can you think of a function $f(x,y)$ where $f_x = f_y = 0$ and $f_{xx}, f_{yy} < 0$ at the origin, but there exists $f(x,y) > f(0,0)$ arbitrarily close to the origin?