## True/False.

1. For a function $f(x, y)$, if $f_x$ and $f_y$ both exists at $(1, 2)$, then for any unit vector $\vec{u} = (a, b)$,

$$D_u f(1, 2) = f_x(1, 2)a + f_y(1, 2)b.$$

2. Consider $f(x_1, x_2, x_3)$.

$$D_v\left(\frac{\partial f}{\partial x_1}\right) = \frac{\partial^2 f}{\partial x_2 \partial x_1} \qquad \text{when} \qquad \vec{v} = \langle 0, 1, 0\rangle.$$

3. If $f(x_1(t), x_2(t), \cdots, x_n(t)) = f(\vec{r}(t))$ is a function of $n$ variables, and each variable $x_i$ is a function of $t$. Then

$$\left.\frac{\partial f}{\partial t}\right|_{t=t_0} = \vec{\nabla} f(t_0) \cdot \vec{r}'(t_0).$$

## Examples.

4. What is the gradient vector of

$$f(x, y, z) = x^2y^4 + \cos(z + y^2) - 7xz?$$

5. Find the directional derivative of

$$g(r, \theta) = r^2 - r\cos(4\theta)$$

in the Cartesian direction of $(-1, 0)$ at $r = 3, \theta = \frac{\pi}{2}$.

6. (a) What is the directional derivative of

$$f(x, y) = \frac{1}{y - x^2} \text{ at } (-2, 3)$$

in the direction of $u = \langle 1, 2, 0\rangle$?

(b) In which direction is the rate of change maximized?

7. Find the normal vector to the surface

$$x^2 + y^2 + z^2 = 65$$

at $(7, 0, 4)$.

8. Find the normal vector to the surface

$$x^3 + yz^2 = 11$$

at $(3, -1, 4)$.