Math 53 DIS 202/204 **Jiazhen Tan (tanjz@berkeley.edu)**
Multivariable Calculus *September 21, 2020*

True/False.

Assume all functions are differentiable.

Here $f(x, y)$ means it maps from $\mathbb{R}^2 \to \mathbb{R}$, and $f(x, y, z)$ means it maps from $\mathbb{R}^3 \to \mathbb{R}$.

1. (a) $D_u f(x, y)$ is maximized when $\vec{u}$ points opposite to $\vec{\nabla} f(x, y)$.
   (b) For $f(x, y)$, there is always a direction $\vec{u}$ where $D_u f(x, y)$ is zero.

2. For $f(x, y, z)$, if $\vec{\nabla} f(x, y, z) \cdot \vec{v} = 0$ for all $\vec{v} \in \mathbb{R}^3$, then f_x, f_y, f_z are all zero there.

3. In a contour map for $f(x, y)$, $\vec{\nabla} f(x, y)$ is parallel to the level curves.

Examples.

4. **Exercise 14.6.40.**

5. One reason we care about directional derivatives is that were often interested in the directional derivative of a function along the normal direction to a curve or surface. These kind of directional derivatives often represent the *flux* through a curve or surface and will reappear in the last third of the course. For example, consider the cylinder S that is the graph of $x^2 + y^2 = 1$ in three-dimensional spacetime (i.e. 2D space with x and y coordinates and 1 time dimension t).

   Let $u(x, y, t)$ be a differentiable function, and let $v(x, y, t)$ denote the normal vector to the surface $x^2 + y^2 = 1$ at a point (x, y, t) on the cylinder.

   (a) Express the cylinder S as the level surface of a function $f(x, y, t)$.
   (b) Use your answer to part (a) above to find the normal vector v at the point and time (x, y, t).
   (c) Write a formula for the directional derivative $D_v u(x, y, t)$ in terms of the function u.

Acknowledgements.

- 1a, 2, 3, 5 are From James Rowan's handout from Summer 2018,

  `https://math.berkeley.edu/~jrowan/53Summer18/MATH53Su18ROWAN-SECTION0710.pdf`