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We assume the 2nd partial derivatives of $f(x, y)$ are continuous on a disk around (a, b).

When $f_x(a, b) = f_y(a, b) = 0$, we define

$$D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - \left(f_{xy}(a, b)\right)^2$$

(a) If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a local minimum.

(b) If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a local maximum.

(c) If $D(a, b) < 0$ then it is neither a local minimum nor a local maximum.

True/False.

1. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then $f_{yy}(a, b) > 0$.
2. If $D_u(D_u f(a, b)) > 0$ in every direction u, then (a, b) is a local minimum.
3. If $D(a, b) = 0$, then f can have a maximum, minimum, or a saddle point at (a, b).

Examples.

4. Give a function with exactly two local maxima, and find all of its critical points.
5. Give a function with exactly one saddle point, and show this is the case.
6. Give a function with more than one saddle point and no local minima or maxima.