



Math 4310
Homework 2
Due 9/5/12

Name: _____

Collaborators: _____

Please print out these pages. I encourage you to work with your classmates on this homework. Please list your collaborators on this cover sheet. (Your grade will not be affected.) Even if you work in a group, you should write up your solutions yourself! You should include all computational details, and proofs should be carefully written with full details. As always, please write neatly and legibly.

Please follow the instructions for the “extended glossary” on separate paper (L^AT_EX it if you can!) Hand in your final draft, including full explanations and write your glossary in complete, mathematically and grammatically correct sentences. Your answers will be assessed for style and accuracy.

Please **staple** this cover sheet, your exercise solutions, and your glossary together, in that order, and hand in your homework in class.

GRADES

Exercises _____ / 50

Extended Glossary

Component	Correct?	Well-written?
Definition	/6	/6
Example	/4	/4
Non-example	/4	/4
Theorem	/5	/5
Proof	/6	/6
Total	/25	/25

Exercises.

- For which natural numbers n is $n^2 \leq 2^n$?
- Let $M_n(\mathbb{R})$ denote the $n \times n$ matrices with coefficients in \mathbb{R} , for $n \geq 1$. This set has two natural operations, matrix addition $+$ and matrix multiplication \cdot . For which n is $M_n(\mathbb{R})$ with these operations a field? Please justify your answer.
- (Curtis [1, p. 25 #1]) In the vector space \mathbb{R}^3 , compute (that is, express in the form (λ, μ, ν)) the following vectors formed from $a = (-1, 2, 1)$, $b = (2, 1, -3)$ and $c = (0, 1, 0)$.
 - $a + b + c$.
 - $2a - b + c$.
 - $-a + 2b$.
 - $\alpha a + \beta b + \gamma c$.
- (Curtis, [1, p. 25 #2]) In \mathbb{R}^3 , with vectors a , b , and c from the previous exercise, solve the equations below for $x = (\alpha_1, \alpha_2, \alpha_3)$ in \mathbb{R}^3 .
 - $x + a = b$.
 - $2x - 3b = c$.
 - $b + x = a - 2c$.
- Show that the set of polynomials $\mathbb{Q}[x]$ in one variable with coefficients in \mathbb{Q} is a vector space over the field \mathbb{Q} . Did your answer depend on \mathbb{Q} , or does your proof generalize to any field \mathbb{F} ?

Extended Glossary. Please give a definition of a **permutation matrix**. Then give an example of a permutation matrix, an example of a matrix that is not a permutation matrix, and state and prove a theorem about permutation matrices.

You probably saw permutation matrices in Math 2210 or 2940. You can find the definition and some properties described on Wikipedia, but the Wikipedia page does not include the full details of the proofs. There is some good advice in our textbook about reading mathematics. Curtis says [1, p. 12] that mathematics “cannot be read like a newspaper article; the reader will find it useful to have paper and pencil handy, and write out the steps, checking the references to the axioms, until the reasoning becomes clear.” Please be sure to write the full details **in your own words** when you write up your extended glossary!

You may work in groups, but please write up your solutions **yourself**. If you do work together, your group should come up with at least two examples, two non-examples, and two theorems. Each one (example/non-example/theorem) should be included in some group member’s extended glossary. Your solutions should be written formally, so that we could cut and paste them into a textbook.

References

- [1] C. Curtis, *Linear Algebra. An Introductory Approach*. Corrected reprint of the 1984 fourth edition. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1993.