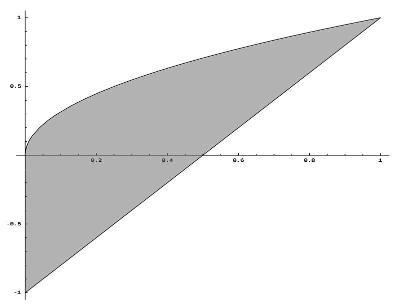
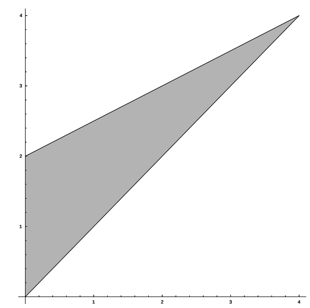
WEEK 2 HW SOLUTIONS

MATH 122



§5.4 # 12



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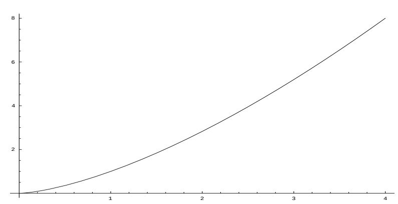

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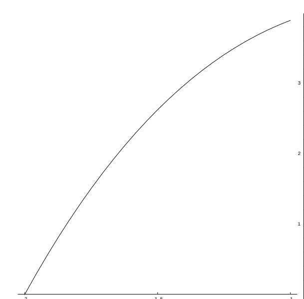

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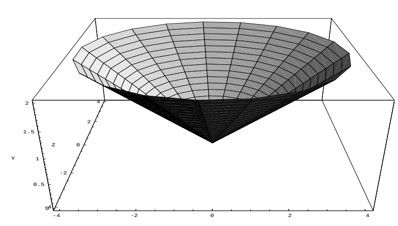
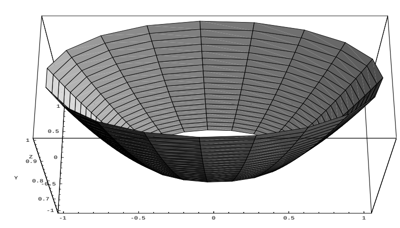


§5.6 # 10



§5.6 # 20

§5.4: Cylindrical Shells

6. For the given graph in the book, we see that $a = 0$ and $b = 3$. Thus, $V = \int_0^3 2\pi(\text{shell radius})(\text{shell height})\,dx = \int_0^3 2\pi x(\frac{9x}{\sqrt{x^3+9}})\,dx$. Under a u-substitution with $u = x^3+9$, this becomes $V = 2\pi\int_9^{36} 3u^{-1/2}\,du = 6\pi[2u^{1/2}]_9^{36} = 12\pi(6-3) = 36\pi$.

12. It's easy to see that $a = 0$, $b = 1$. Thus, $V = \int_0^1 2\pi(\text{shell radius})(\text{shell height})\,dx = \int_0^1 2\pi x(\sqrt{x} - (2x-1))\,dx = 2\pi\int_0^1 x^{3/2} - 2x^2 + x\,dx = 2\pi[\frac{2x^{5/2}}{5} - \frac{2x^3}{3} + \frac{x^2}{2}]_0^1 = 2\pi(\frac{2}{5} - \frac{2}{3} + \frac{1}{2}) = \frac{7\pi}{15}$

28. (a) Use the washer method: $V = \int_0^4 \pi[R(x)^2 - r(x)^2]\,dx = \pi\int_0^4 (\frac{x^2}{2}+2)^2 - x^2\,dx = \pi\int_0^4 -\frac{3}{4}x^2 + 2x + 4\,dx = \pi[-\frac{x^3}{4} + x^2 + 4x]_0^4 = \pi(-16+16+16) = 16\pi$.

(b) Use the shell method: $V = \int_0^4 2\pi(\text{shell radius})(\text{shell height})\,dx = \int_0^4 2\pi x(\frac{x}{2} + 2 - x)\,dx = 2\pi\int_0^4 2x - \frac{x^2}{2}\,dx = 2\pi[x^2 - \frac{x^3}{6}]_0^4 = 2\pi(16 - \frac{64}{6}) = \frac{32\pi}{3}$

(c) Use the shell method: $V = \int_0^4 2\pi(\text{shell radius})(\text{shell height})\,dx = \int_0^4 2\pi(4 - x)(\frac{x}{2}+2-x)\,dx = 2\pi\int_0^4 8-4x+\frac{x^2}{2}\,dx = 2\pi[8x-2x^2+\frac{x^3}{6}]_0^4 = 2\pi(32-32+\frac{64}{6}) = \frac{64\pi}{3}$

(d) Use the washer method: $V = \int_0^4 \pi[R(x)^2 - r(x)^2]\,dx = \pi\int_0^4 (8-x)^2 - (6 - \frac{x}{2})^2\,dx = \pi\int_0^4 \frac{3x^2}{4} - 10x + 28\,dx = \pi[\frac{x^3}{4} - 5x^2 + 28x]_0^4 = 48\pi$

§5.5: Lengths of Plane Curves

10. First, $\frac{dy}{dx} = \frac{3x^{1/2}}{2}$. So, $L = \int_0^4 \sqrt{1 + \frac{9x}{4}}\,dx$. Using a u-substitution with $u = 1+\frac{9x}{4}$, this becomes $L = \int_1^{10} \frac{4}{9}u^{1/2}\,du = \frac{4}{9}[\frac{2u^{3/2}}{3}]_1^{10} = \frac{8}{27}(10\sqrt{10} - 1)$.

18. By the Fundamental Theorem of Calculus, $\frac{dy}{dx} = \sqrt{3x^4 - 1}$. Thus, $L = \int_{-2}^{-1} \sqrt{1 + (3x^4 - 1)}\,dx = \int_{-2}^{-1} \sqrt{3}x^2\,dx = \sqrt{3}[\frac{x^3}{3}]_{-2}^{-1} = \frac{\sqrt{3}}{3}(-1+8) = \frac{7\sqrt{3}}{3}$.

§5.6: Areas of Surfaces of Revolution

10. We're rotating about the y-axis, so we solve for x and find $\frac{dx}{dy}$: $x = 2y$, so $\frac{dx}{dy} = 2$. Hence, $S = \int_0^2 2\pi 2y\sqrt{1+2^2}\,dy = 4\pi\sqrt{5}\int_0^2 y\,dy = 8\pi\sqrt{5}$. Based on the geometric formula, $S = \frac{1}{2}(\text{base circumference})(\text{slant height}) = \frac{1}{2}(8\pi)(\sqrt{4^2+2^2}) = 4\pi(2\sqrt{5}) = 8\pi\sqrt{5}$, which agrees with the integral (phew!).

20. Again, we're rotating about the y-axis. This time, $\frac{dx}{dy} = -\frac{1}{\sqrt{2y-1}}$. Thus, $S = \int_{5/8}^1 2\pi\sqrt{2y-1}\sqrt{1 + \frac{1}{2y-1}}\,dy = 2\pi\int_{5/8}^1 \sqrt{(2y-1)+1}\,dy = 2\pi\int_{5/8}^1 \sqrt{2y}\,dy = 2\pi\sqrt{2}[\frac{2y^{3/2}}{3}]_{5/8}^1 = \frac{4\pi\sqrt{2}}{3}(1 - \frac{5\sqrt{5}}{8\sqrt{8}}) = \frac{\pi}{12}(16\sqrt{2} - 5\sqrt{5})$.

28. Using the labels on the picture in the book, we will slice the loaf of radius r at $x = a$ and $x = a+h$, yielding a yummy piece of bread h units wide. Assuming that

nothing stupid happens (i.e., we assume that $a \geq -r$ and $a + h \leq r$; otherwise you would be cutting where there is no bread), we can find the surface area of our slice using the formulae from this section. Thus, $y = \sqrt{r^2 - x^2}$, so $\frac{dy}{dx} = -\frac{x}{\sqrt{r^2-x^2}}$. Hence, $S = \int_a^{a+h} 2\pi\sqrt{r^2 - x^2}\sqrt{1 + \frac{x^2}{r^2-x^2}}\,dx = 2\pi \int_a^{a+h} \sqrt{(r^2 - x^2) + x^2}\,dx = 2\pi r \int_a^{a+h} dx = 2\pi rh$, which doesn't depend on a. Cool!