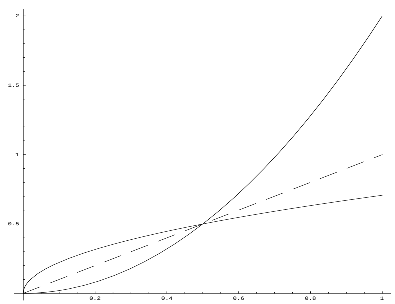
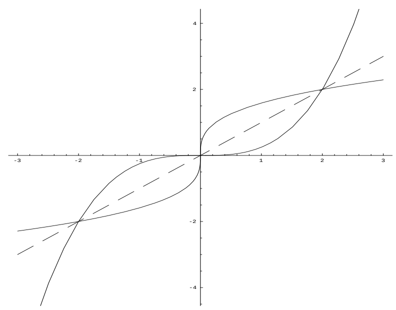
WEEK 3 HW SOLUTIONS

MATH 122



§6.1 # 28



§6.1 # 30

§6.1: Inverse Functions and Their Derivatives

18. $f(x) = x^{2/3}$ and $x \geq 0$, so $y = x^{2/3} \Rightarrow y^{3/2} = x$. Thus, $f^{-1}(x) = x^{3/2}$.

28. $f(x) = 2x^2$, $x \geq 0$ and $a = 5$. $y = 2x^2 \Rightarrow \frac{y}{2} = x^2 \Rightarrow \sqrt{\frac{y}{2}} = x$, (ONLY since $x \geq 0$) so $f^{-1}(x) = \sqrt{\frac{x}{2}}$. Then, $f'(5) = 4(5) = 20$ and $(f^{-1})'(f(5)) = \frac{1}{2\sqrt{2f(5)}} = \frac{1}{2\sqrt{100}} = \frac{1}{20}$.

30. a) $h(k(x)) = h((4x)^{1/3}) = \frac{((4x)^{1/3})^3}{4} = \frac{4x}{4} = x$ and $k(h(x)) = k(\frac{x^3}{4}) = (4\frac{x^3}{4})^{1/3} = (x^3)^{1/3} = x$.
b) see graph above
c) $h'(x) = \frac{3x^2}{4}$, so $h'(2) = 3$ and $h'(-2) = 3$. Also, $k'(x) = \frac{4}{3(4x)^{2/3}}$, so $k'(2) = \frac{1}{3}$ and $k'(-2) = \frac{1}{3}$.
d) $h'(0) = 0$, so the line $y = 0$ is the tangent line. However, $\lim_{x \to 0} k'(x) = \infty$, so the line $x = 0$ is the tangent line. (Note the limit convention being expressed here: $\frac{1}{0} = \infty$)

§6.6: L'Hôpital's Rule

8. $\lim_{t \to 0} \frac{\sin(5t)}{t}$ is indeterminate of the form $\frac{0}{0}$, so, by L'Hôpital's, this is equal to $\lim_{t \to 0} \frac{5\cos(5t)}{1} = 5$.

12. $\lim_{\theta \to -\frac{\pi}{3}} \frac{3\theta + \pi}{\sin(\theta + \frac{\pi}{3})}$ is again of the form $\frac{0}{0}$, so is equal to $\lim_{\theta \to -\frac{\pi}{3}} \frac{3}{\cos(\theta + \frac{\pi}{3})} = 3$.

24. $\lim_{x \to 0} \frac{3^x - 1}{2^x - 1}$ is of the $\frac{0}{0}$ form, so is equal to $\lim_{x \to 0} \frac{3^x \ln(3)}{2^x \ln(2)} = \frac{\ln(3)}{\ln(2)}$.

38. $\lim_{x \to \infty} \frac{1}{x \ln(x)} \int_1^x \ln(t)\, dt$ is of the form $\frac{\infty}{\infty}$, so is equal to $\lim_{x \to \infty} \frac{\ln(x)}{\ln(x) + 1}$, which is again of the form $\frac{\infty}{\infty}$, so is equal to $\lim_{x \to \infty} \frac{1/x}{1/x} = 1$.

50. $\lim_{x\to 0}(e^x + x)^{1/x}$ is indeterminate of the form 1^∞. We solve this by letting $y = (e^x + x)^{1/x}$ and looking at $\lim_{x\to 0} \ln(y) = \lim_{x\to 0} \frac{\ln(e^x+x)}{x}$. This is of the form $\frac{0}{0}$, so we use l'Hôpital's rule to find that it equals $\lim_{x\to 0} \frac{e^x+1}{e^x+x} = 2$. Thus, the actual limit we want (that of y, not of $\ln(y)$) is e^2.

54. As the book states, this can't be done with l'Hôpital's rule. However, we'll use the same sort of trick as we did in # 50; that is, if $f(x)$ is a continuous function, then $\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$ (we used this fact in # 50 with $f(x) = \ln(x)$). In this case, $f(x) = \sqrt{x}$. So, $\lim_{x\to 0^+} \sqrt{\frac{x}{\sin(x)}} = \sqrt{\lim_{x\to 0^+} \frac{x}{\sin(x)}}$. The limit inside is of the form $\frac{0}{0}$, so we can use l'Hôpital's again and get $\sqrt{\lim_{x\to 0^+} \frac{1}{\cos(x)}} = \sqrt{1} = 1$.