WEEK 4 HW SOLUTIONS

MATH 122

§6.8: Inverse Trigonometric Functions

18. $\sec(\cos^{-1}(\frac{1}{2})) = \sec(\frac{\pi}{3}) = 2$

24. $\cot(\sin^{-1}(-\frac{1}{2}) - \sec^{-1}(2)) = \cot(-\frac{\pi}{6} - \frac{\pi}{3}) = \cot(-\frac{\pi}{2}) = 0.$

44. $\lim_{x\to-\infty} \tan^{-1}(x) = -\frac{\pi}{2}$ (think about it).

50. We'll use the washer method. In that case, the outer radius $R(y) = 2$ and the inner radius $r(y) = \sec(y)$. Thus, the volume is $V = \int_0^{\pi/3} \pi(2^2 - \sec^2(y))\, dy = \pi[4y - \tan(y)]_0^{\pi/3} = \pi[\frac{4\pi}{3} - \sqrt{3}]$.

§6.9: Derivatives and Integrals of Inverse Trigonometric Functions

14. $y = \tan^{-1}(\ln(x))$. Then $\frac{dy}{dx} = \frac{1}{1+(\ln(x))^2} \cdot \frac{1}{x} = \frac{1}{x+x(\ln(x))^2}$.

28. $\int \frac{dx}{x\sqrt{5x^2-4}} = \int \frac{du}{u\sqrt{u^2-4}} = \frac{1}{2}\sec^{-1}|\frac{u}{2}| + C = \frac{1}{2}\sec^{-1}(\frac{\sqrt{5}}{2}|x|) + C$, where we used a u-substitution with $u = \sqrt{5}x$.

66. $\lim_{x\to1^+} \frac{\sqrt{x^2-1}}{\sec^{-1}(x)} = \lim_{x\to1^+} \frac{x(x^2-1)^{-1/2}}{\frac{1}{|x|\sqrt{x^2-1}}} = \lim_{x\to1^+} x^2 = 1$. Note that $|x| = x$ since we're only considering x's near 1.

74. $\frac{dy}{dx} = \frac{1}{1+x^2} - 1$ and $y(0) = 1$. So, we integrate to find that $y = \tan^{-1}(x) - x + C$. Checking our inital condition, $y(0) = \tan^{-1}(0) - 0 + C = C$, so $C = 1$. Therefore, $y = \tan^{-1}(x) - x + 1$.

§6.10: Hyperbolic Functions

36. $y = \cosh^{-1}(\sec(x))$, so $\frac{dy}{dx} = \frac{1}{\sqrt{\sec^2(x)-1}} \cdot \sec(x)\tan(x) = \frac{\sec(x)\tan(x)}{|\tan x|} = \sec(x)$ on $0 < x < \frac{\pi}{2}$.

40. $y = x\tanh^{-1}(x) + \frac{1}{2}\ln(1 - x^2) + C$. Then $\frac{dy}{dx} = \tanh^{-1}(x) + \frac{x}{1-x^2} + \frac{1}{2}(\frac{-2x}{1-x^2}) = \tanh^{-1}(x)$, so the formula is correct.

52. $\int_0^{\ln(2)} \tanh(2x)\, dx = \int_0^{\ln(2)} \frac{\sinh(2x)}{\cosh(2x)}\, dx = \frac{1}{2}\int_1^{17/8} \frac{du}{u} = \frac{1}{2}\ln(\frac{17}{8})$. We used a u-substitution with $u = \cosh(x)$. Our bounds come from the fact that $\cosh(0) = 1$ and $\cosh(2\ln(2)) = \cosh(\ln(4)) = \frac{e^{\ln(4)}+e^{-\ln(4)}}{2} = \frac{17}{8}$.

60. $\int_0^{\ln(10)} 4\sinh^2(\frac{x}{2})\, dx = \int_0^{\ln(10)} 4(\frac{\cosh(x)-1}{2})\, dx = 2\int_0^{\ln(10)} \cosh(x) - 1\, dx = 2[\sinh(x) - x]_0^{\ln(10)} = e^{\ln(10)} - e^{-\ln(10)} - 2\ln(10) = 9.9 - 2\ln(10)$.