HW 5 SOLUTIONS

MATH 122

§7.2: Integration by Parts

16. The tabular method is the easiest way to do this. Here's the table (since I can't draw diagonal arrows, I've shifted the first column down a bit):

		e^{4t}
t^2	$(+)$	$\frac{1}{4}e^{4t}$
$2t$	$(-)$	$\frac{1}{16}e^{4t}$
2	$(+)$	$\frac{1}{64}e^{4t}$
0		

Thus, $\int t^2e^{4t}\,dt = e^{4t}(\frac{t^2}{4} - \frac{t}{8} + \frac{1}{32}) + C$.

24. Here we have to use the integration by parts formula in a creative way. Using $dv = e^{-2x}dx$ and $u = \sin(2x)$,

$$\begin{aligned}\int e^{-2x}\sin(2x)\,dx &= -\frac{1}{2}e^{-2x}\sin(2x) - \int -\frac{1}{2}e^{-2x}(2\cos(2x))\,dx \\ &= -\frac{1}{2}e^{-2x}\sin(2x) + \int e^{-2x}\cos(2x)\,dx.\end{aligned}$$

Repeating the integration by parts, this time with $dv = e^{-2x}dx$ and $u = \cos(2x)$, we end up with

$$\begin{aligned}\int e^{-2x}\sin(2x)\,dx &= -\frac{1}{2}e^{-2x}\sin(2x) + -\frac{1}{2}e^{-2x}\cos(2x) - \int -\frac{1}{2}e^{-2x}(-2\sin(2x))\,dx \\ &= -\frac{e^{-2x}}{2}(\sin(2x)+\cos(2x)) - \int e^{-2x}\sin(2x)\,dx.\end{aligned}$$

Thus, adding the quantity $\int e^{-2x}\sin(2x)\,dx$ to both sides of the equation, we get

$$2\int e^{-2x}\sin(2x)\,dx = -\frac{e^{-2x}}{2}(\sin(2x)+\cos(2x)).$$

Therefore, $\int e^{-2x}\sin(2x)\,dx = -\frac{e^{-2x}}{4}(\sin(2x)+\cos(2x))$.

28. Using $u = \ln(x+x^2)$ and $dv = dx$, we find that

$$\begin{aligned}\int \ln(x+x^2)\,dx &= x\ln(x+x^2) - \int x\frac{2x+1}{x+x^2}\,dx \\ &= x\ln(x+x^2) - \int \frac{2x+1}{1+x}\,dx \\ &= x\ln(x+x^2) - \int \frac{2(x+1)-1}{x+1}\,dx \\ &= x\ln(x+x^2) - \int 2\,dx + \int \frac{1}{x+1}\,dx \\ &= x\ln(x+x^2) - 2x + \ln|x+1| + C.\end{aligned}$$

§7.3: Partial Fractions

18. First, we'll reduce the fraction: $\frac{x^3}{x^2-2x+1} = x+2+\frac{3x-2}{x^2-2x+1}$, by long division. Now, we'll find the partial fraction decomposition for the remainder term: $\frac{3x-2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$, so $3x-2 = Ax - A + B$, that is, $A = 3$ and $B = 1$. Then,

$$\begin{aligned}\int_{-1}^{0} \frac{x^3}{(x-1)^2}\,dx &= \int_{-1}^{0} x+2+\frac{3}{x-1}+\frac{1}{(x-1)^2}\,dx \\ &= \left[\frac{x^2}{2} + 2x + 3\ln|x-1| - \frac{1}{x-1}\right]_{-1}^{0} \\ &= 1 - (\frac{1}{2} - 2 + 3\ln(2) + \frac{1}{2}) = 2 - 3\ln(2).\end{aligned}$$

26. $\frac{s^4+81}{s(s^2+9)^2}$ is already reduced, since the power of s in the denominator is 5. So, we get to have lots of partial fraction fun! $\frac{s^4+81}{s(s^2+9)^2} = \frac{A}{s} + \frac{Bs+C}{s^2+9} + \frac{Ds+E}{(s^2+9)^2}$. This leads to

$$s^4 + 81 = (A+B)s^4 + Cs^3 + (18A+9B+D)s^2 + (9C+E)s + 81A$$

From the constant term, we deduce that $A = 1$. From the s^4-term, we get that $A + B = 1$, whence $B = 0$. From the s^3-term, we see that $C = 0$. The s^2-term gives us $18A + 9B + D = 0$, so $D = -18$ and the s-term gives us $E = 0$. So, we have that $\frac{s^4+81}{s(s^2+9)^2} = \frac{1}{s} - \frac{18s}{(s^2+9)^2}$. Thus, $\int \frac{s^4+81}{s(s^2+9)^2}\,ds = \int \frac{1}{s} - \frac{18s}{(s^2+9)^2}\,ds = \ln|s| - 9\int \frac{du}{u^2} = \ln|s| + \frac{9}{s^2+9} + C$, where I used the substitution $u = s^2+9$.

32. From long division, we get that $\frac{16x^3}{4x^2-4x+1} = 4x+4+\frac{12x-4}{4x^2-4x+1}$. The denominator is just $(2x-1)^2$, so using the partial fraction method, $12x-4 = 2Ax - A + B$, whence $A = 6$ and $B = 2$. Then $\int \frac{16x^3}{4x^2-4x+1}\,dx = \int 4x+4+\frac{6}{2x-1}+\frac{2}{(2x-1)^2}\,dx = 2x^2 + 4x + 3\ln|2x+1| - \frac{1}{2x-1} + C$.

§7.4: Trig. substitutions

10. $\int \frac{5}{\sqrt{25x^2-9}}\,dx = \int \frac{dx}{\sqrt{x^2-\frac{9}{25}}}$, so we'll use the trig. substitution $x = \frac{3}{5}\sec(\theta)$, which gives us $dx = \frac{3}{5}\sec(\theta)\tan(\theta)d\theta$. Then the integral becomes

$$\begin{aligned}\int \frac{5}{\sqrt{25x^2-9}}\,dx &= \int \frac{\frac{3}{5}\sec(\theta)\tan(\theta)d\theta}{\frac{3}{5}\sqrt{\tan^2(\theta)}} \\ &= \int \sec(\theta)\,d\theta \\ &= \int \frac{\sec^2(\theta)+\sec(\theta)\tan(\theta)d\theta}{\sec(\theta)+\tan(\theta)} \\ &= \ln|\sec(\theta)+\tan(\theta)|+C \\ &= \ln|\frac{5}{3}x+\sqrt{\frac{25}{9}x^2-1}|+C.\end{aligned}$$

20. Using the trig. substitution $x = 2\sin(\theta)$ (with $dx = 2\cos(\theta)d\theta$), we see that $\theta = \arcsin(\frac{x}{2})$. Thus, when $x = 0$, $\theta = 0$ and when $x = 1$, $\theta = \frac{\pi}{6}$. Then,

$$\begin{aligned}\int_0^1 \frac{dx}{(4-x^2)^{3/2}} &= \int_0^{\pi/6} \frac{2\cos(\theta)d\theta}{(4\cos^2(\theta))^{3/2}} \\ &= \int_0^{\pi/6} \frac{2\cos(\theta)d\theta}{8\cos^3(\theta)} \\ &= \int_0^{\pi/6} \frac{d\theta}{4\cos^2(\theta)} \\ &= \frac{1}{4}\int_0^{\pi/6} \sec^2(\theta)\,d\theta \\ &= \frac{1}{4}[\tan(\theta)]_0^{\pi/6} \\ &= \frac{1}{4\sqrt{3}}.\end{aligned}$$

26. Since $\int \frac{6dt}{(9t^2+1)^2} = \int \frac{6dt}{81(t^2+\frac{1}{9})^2}$, we use $t = \frac{1}{3}\tan(\theta)$ and $dt = \frac{1}{3}\sec^2(\theta)d\theta$. This gives us

$$\begin{aligned}\int \frac{6dt}{(9t^2+1)^2} &= \int \frac{2\sec^2(\theta)d\theta}{81(\frac{1}{9}\sec^2(\theta))^2} \\ &= \int \frac{2d\theta}{\sec^2(\theta)} \\ &= \int 2\cos^2(\theta)\,d\theta \\ &= \theta+\sin(\theta)\cos(\theta)+C \\ &= \arctan(3t)+\frac{3t}{9t^2+1}+C.\end{aligned}$$

44. Under the given substitution,

$$\begin{aligned}
\int \frac{dx}{1+\sin(x)+\cos(x)} &= \int \frac{\frac{2dz}{1+z^2}}{1+\frac{2z}{1+z^2}+\frac{1-z^2}{1+z^2}} \\
&= \int \frac{2dz}{1+z^2+2z+1-z^2} \\
&= \int \frac{2dz}{2+2z} \\
&= \int \frac{dz}{1+z} \\
&= \ln|1+z|+C \\
&= \ln|1+\tan(\frac{x}{2})|+C.
\end{aligned}$$