Math 105 Prelim #2 – October 28, 2004

This exam has a formula sheet, 7 problems and 7 numbered pages.

You have 90 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

Calculators are permitted, but no other aids are allowed.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ______________________________

Instructor: ______________________________

Problem	Score
1	
2	
3	
4	
5	
6	
7	

TOTAL: __________

Formula Sheet

Definition

$$P(E) = \frac{\mathrm{n}(E)}{\mathrm{n}(S)}, \qquad \text{only for equally likely outcomes.}$$

Union

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Complement

$$1 = P(E) + P(E')$$

Mutually Exclusive Events

$$P(E \cup F) = P(E) + P(F)$$

Conditional Probability

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\mathrm{n}(E \cap F)}{\mathrm{n}(F)}$$

Independent Events

$$\begin{aligned} P(E \cap F) &= P(E) \cdot P(F), \quad \text{or} \\ P(E|F) &= P(E), \quad \text{or} \\ P(F|E) &= P(F) \end{aligned}$$

Bayes' Theorem

$$P(F_i|E) = \frac{P(F_i) \cdot F(E|F_i)}{P(F_1) \cdot F(E|F_1) + P(F_2) \cdot F(E|F_2) + \cdots + P(F_n) \cdot F(E|F_n)}$$

Bayes' Theorem (Special Case)

$$P(F|E) = \frac{P(F) \cdot P(E|F)}{P(F) \cdot P(E|F) + P(F') \cdot P(E|F')}$$

Probability Distribution Let $S = \{s_1, \ldots, s_n\}$. Then $p_i = P(\{s_i\})$ is a valid probability distribution on S if the following two conditions hold.

(i) $0 \leq p_1 \leq 1, 0 \leq p_2 \leq 1, \ldots, 0 \leq p_n \leq 1$

(ii) $p_1 + p_2 + \cdots + p_n = 1$

Counting

$$P(n, r) = \frac{n!}{(n-r)!} = n(n-1)\cdots(n-r+1) \qquad \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

1. (*10 points*) José has a wine collection consisting of 5 bottles of red wine and 7 bottles of white wine (each bottle is different).

(a) If Henri is going to steal 3 bottles from José, how many choices does Henri have?

(b) In how many ways can Henri steal 3 bottles if he takes precisely one bottle of red wine?

2. (*12 points*) Suppose that we choose a number at random between 0 and 99 (0 and 99 are possible). Let E be the event that we get a number less than or equal to 19, and let F be the event that we get a number whose digits sum to 9.

(a) Write down the events E and F. What are $P(E)$ and $P(F)$?

(b) Show that E and F are independent events, or show that E and F are not independent events.

(c) Let G be the event that we get a number whose digits sum to 5. Are E and G independent events?

3. (*16 points*) Suppose that we have three jars. The first jar contains 3 black balls and 7 white balls, the second jar contains 2 black balls and 3 white balls, and the third jar contains 1 black ball and 1 white ball. Consider the following experiment: We roll a standard 6-sided die. If the die shows 1, we take a ball from the first jar, if the die shows 2 or 3, we take a ball from the second jar, and if the die shows 4, 5, or 6, we take a ball from the third jar.

(a) Draw a tree diagram for this experiment.

(b) What is the probability that we get a black ball?

(c) We asked our friend Bob to do this experiment and he got a black ball. What is the probability that he got this black ball from the second jar? What is the probability that the die showed a 2?

4. (*14 points*)

In a certain presidential election, candidate "Erwin Addison" received 51% of the vote, and candidate "Morris Liberty" received the other 49% of the vote. 58% of those who voted for Mr. Addison were women, and 45% of those who voted for Mr. Liberty were women. A voter is chosen uniformly at random. Let A be the event "voter voted for Mr. Addison", let L be the event "voter voted for Mr. Liberty", and let F be the event "voter is female".

(a) What is the probability of the event A? What is the conditional probability of the event F given A?

(b) Using Bayes' Theorem, find the probability that the voter voted for Mr. Addison given that the voter is a woman.

(c) What percentage of voters in this election were women?

5. (*16 points*) Henri has a collection of 9 different books.

(a) In how many ways can Henri arrange these books on his shelf?

(b) The collection in fact consists of 3 math books, 4 physics books and 2 chemistry books. If books on the same subject must be grouped together, how many arrangements are possible?

(c) Henri has 4 friends: Antonio, Drew, José and Laurent. In how many ways can Henri give each of his friends one book from his collection?

(d) How many ways are there to do this if Henri gives away at least one math book?

6. (*12 points*) In the card game "Pirate's Bluff", a player is dealt 4 cards from a standard deck of 52. A famous hand in this game is known as a "Jack in the Basement". It consists of exactly one Jack, along with a pair of one value, and a single of a different value (for example: one Jack, two 8's and one Queen). Given that $\binom{52}{4} = 270,725$ what is the probability of being dealt a "Jack in the Basement"?

Recall that in a standard deck of 52 *cards each card has two attributes: a value and a suit. There are four possible suits: hearts, clubs, diamonds and spades. There are thirteen possible values: Ace, 2, ... , 10, Jack, Queen, King.*

7. (*20 points*) Professor Thaddeus McGoo has an urn on his desk containing 15 marbles: 5 red, 5 blue, and 5 green. He asks his student Billy to grab 6 marbles from the urn and place them on his desk.

(a) What is the probability that Billy grabs 3 red marbles, 2 blue marbles, and 1 green marble?

(b) What is the probability that exactly 4 of Billy's marbles are blue?

(c) What is the probability that Billy grabs at least 4 blue marbles?

(d) What is the probability that exactly 4 of Billy's marbles are the same color?