$\boxed{\textbf{MATH 105: Homework 1 Solutions}}$

Each problem worth 4 points

1.1.20
Find the equation $y = mx + b$ of the line through $(8, -1)$ and $(4, 3)$.
The slope is given by rise over run:

$$m = \frac{3 - (-1)}{4 - 8} = \frac{4}{-4} = -1.$$

Then we plug in the point $(4, 3)$ to find b:

$$\begin{aligned} y &= mx + b \\ 3 &= (-1) \cdot 4 + b \\ 7 &= b. \end{aligned}$$

Hence the equation is $y = -x + 7$.

□

1.1.26
Find the equation $y = mx + b$ of the line with x-intercept -2 and y-intercept 4.
We know that the line goes through $(-2, 0)$ and $(0, 4)$. Plugging these points in, we get two equations for m and b:

$$\begin{aligned} 0 &= -2m + b, \\ 4 &= 0m + b. \end{aligned}$$

Substituting $b = 4$ into the first equation gives $m = 2$. Hence the equation is $y = 2x + 4$.

□

1.1.36
Find k so that the line through $(4, -1)$ and $(k, 2)$ is

(a) parallel to $2x + 3y = 6$,

(b) perpendicular to $5x - 2y = -1$.

The slope of the line through $(4, -1)$ and $(k, 2)$ is

$$\frac{2 - (-1)}{k - 4} = \frac{3}{k - 4}.$$

(a) Now, note that the slope of the line $2x+3y=6$ is $-\frac{2}{3}$, since we may rewrite the equation as $y=-\frac{2}{3}x+3$. Since parallel lines have the same slope, we want

$$\begin{aligned} -\frac{2}{3} &= \frac{3}{k-4} \\ 2(k-4) &= -3\cdot 3 \\ 2k-8 &= -9 \\ 2k &= -1 \\ k &= -\frac{1}{2}. \end{aligned}$$

(b) The slope of the line $5x-2y=-1$ is $\frac{5}{2}$ since we may rewrite the equation as $y=\frac{5}{2}x+\frac{1}{2}$. The product of the slopes of perpendicular lines is -1. Hence we want

$$\begin{aligned} \frac{5}{2}\cdot\frac{3}{k-4} &= -1 \\ \frac{15}{2k-8} &= -1 \\ 15 &= -2k+8 \\ 2k &= -7 \\ k &= -\frac{7}{2}. \end{aligned}$$

$\square$

1.2.22

(a) Let the cost function be $C(x)=mx+b$. We are given that the marginal cost (the cost to produce each item) is \$3.50, and the cost to produce 60 shirts is \$300. To find the fixed cost b, we substitute 60 into $C(x)$:

$$\begin{aligned} C(60) = (3.5)\cdot 60 + b &= 300 \\ 210 + b &= 300 \\ b &= 90. \end{aligned}$$

Hence the cost function is $C(x)=3.5x+90$.

(b) She charges a price of $p=\$9$ for each shirt. Hence the revenue function is $R(x)=9x$. To find the break even quantity, we equate cost and revenue:

$$\begin{aligned} R(x) &= C(x) \\ 9x &= 3.5x + 90 \\ 5.5x &= 90 \\ x &= 90/5.5 = 16.36. \end{aligned}$$

She must sell 17 shirts to break even.

(c) The profit is revenue minus cost $P(x) = R(x) - C(x)$. We want to solve for x when $P(x) = 500$:

$$\begin{aligned} P(x) &= 500 \\ 9x - (3.5x + 90) &= 500 \\ 5.5x - 90 &= 500 \\ 5.5x &= 590 \\ x &= 590/5.5 = 107.27. \end{aligned}$$

She must sell 108 shirts to make a profit of \$500.

□

1.2.28

We are given the cost and revenue functions $C(x) = 12x + 39$ and $R(x) = 25x$.

(a) To find the break even quantity, equate $C(x)$ and $R(x)$:

$$\begin{aligned} R(x) &= C(x) \\ 25x &= 12x + 39 \\ 13x &= 39 \\ x &= 3. \end{aligned}$$

The break even quantity is $x = 3$.

(b) Evaluate the profit $P(x) = R(x) - C(x)$ at $x = 250$:

$$\begin{aligned} P(250) &= 25(250) - 12(250) - 39 \\ &= 6250 - 3000 - 39 \\ &= 3211. \end{aligned}$$

The profit from selling 250 items is \$3211.

(c) Equate the profit to \$130 and solve for x:

$$\begin{aligned} P(x) = 25x - 12x - 39 &= 130 \\ 13x - 39 &= 130 \\ 13x &= 169 \\ x &= 169/13 = 13. \end{aligned}$$

To make a profit of \$130, we must sell 13 units.

□

1.3.14
Let x be the year, with $x = 0$ corresponding to 1970, and let y be the poverty level income cutoff in thousands. We are given that $\Sigma x = 105$, $\Sigma x^2 = 2275$, $\Sigma y = 75.402$, $\Sigma y^2 = 968.270792$, $\Sigma xy = 1460.97$.

(a) Yes, the data appear to lie along a straight line.

(b) The coefficient of correlation is

$$r = \frac{7(1460.97) - (105)(75.402)}{\sqrt{7(2275) - (105)^2} \cdot \sqrt{7(968.270792) - (75.402)^2}} \approx .998$$

There is a strong positive linear correlation between income and year.

(c) The least squares line is $Y = mx + b$ with m and b given by

$$\begin{aligned} 7b + 105m &= 75.402, \\ 105b + 2275m &= 1460.97. \end{aligned}$$

Solving for b in the first equation gives $b = (75.402 - 105m)/7$. Then plugging into the second equation, we get

$$\begin{aligned} 105\left(\frac{75.402 - 105m}{7}\right) + 2275m &= 1460.97 \\ 105(75.402 - 105m) + 15,925m &= 10,226.79 \\ 7917.21 - 11,025m + 15,925m &= 10,226.79 \\ 4900m &= 2309.58 \\ m &\approx .471, \end{aligned}$$

and hence $b = (75.402 - 105(.471))/7 \approx 3.702$.

So, the least squares line is $Y = .471x + 3.702$.

(d) The year 2015 corresponds to $x = 45$. Plugging into the least squares line, we get

$$Y = .471(45) + 3.702 \approx 24.897.$$

The predicted poverty level in the year 2015 is \$24,897.

□

1.3.18

(a) Let x represent the year after 1900 and let y be the men's record. From the table on page 42, we calculate $\Sigma x = 500$, $\Sigma y = 1070.04$, $\Sigma xy = 52,302.5$, $\Sigma x^2 = 33,250$, $\Sigma y^2 = 114,677.325$, with $n = 10$ data points. Now we solve for m and b:

$$\begin{aligned} 10b + 500m &= 1070.04 \\ 500b + 33,250m &= 52,302.5 \\ 10b &= 1070.04 - 500m \\ b &= 107.004 - 50m \end{aligned}$$

$$\begin{aligned} 500(107.004 - 50m) + 33,250m &= 52,302.5 \\ 53,502 - 25,000m + 33,250m &= 52,302.5 \\ 8250m &= -1199.5 \\ m &= -.1454 \end{aligned}$$

$$\begin{aligned} b &= 107.004 - 50(-.1454) \\ &\approx 114.27. \end{aligned}$$

So the least squares line is $Y = -.1454x + 114.27$.

(b) Let x represent the year after 1900 and let y be the women's record. From the table on page 42, we calculate $\Sigma x = 480$, $\Sigma y = 998.64$, $\Sigma xy = 58,032.4$, $\Sigma x^2 = 33,000$, $\Sigma y^2 = 125,562.6272$, with $n = 8$ data points.

Now we solve for m and b:

$$\begin{aligned} 8b + 480m &= 998.64 \\ 480b + 33,000m &= 58,032.4 \\ 8b &= 998.64 - 480m \\ b &= 124.83 - 60m \end{aligned}$$

$$\begin{aligned} 480(124.83 - 60m) + 33,000m &= 58,032.4 \\ 59,918.4 - 28,800m + 33,000m &= 58,032.4 \\ 4200m &= -1886 \\ m &= -.4499 \end{aligned}$$

$$\begin{aligned} b &= 124.83 - 60(-.4499) \\ &= 151.77. \end{aligned}$$

The least squares line is $Y = -.4499x + 151.77$.

(c) Equate the least squares lines for the women and the men:

$$\begin{aligned} -.1454x + 114.27 &= -.4499x + 151.77 \\ .3036x &= 37.5 \\ x &\approx 124 \end{aligned}$$

$$1900 + 124 = 2024.$$

The women will catch up to the men in 2024.

(d)

$$r_{\text{men}} = \frac{10(52,302.5) - (500)(1070.04)}{\sqrt{10(33,250) - (500)^2} \cdot \sqrt{10(114,677.325) - (1070.04)^2}} = -.9877$$

$$r_{\text{women}} = \frac{8(58,032.4) - (480)(998.64)}{\sqrt{8(33,000) - (480)^2} \cdot \sqrt{8(125,562.6272) - (998.64)^2}} = -.9688$$

Both sets of points closely fit a line with negative slope.

□

2.1.8
Solve the system of two linear equations:

$$\begin{aligned} 4m + 3n &= -1, \\ 2m + 5n &= 3. \end{aligned}$$

First, eliminate m from the second equation.

$$\begin{array}{rr} R_1 & 4m+3n=-1 \\ R_1+(-2)R_2 \to R_2 & -7n=-7 \end{array}$$

Then, make each leading coefficient equal to 1.

$$\begin{array}{rc} \frac{1}{4}R_1 \to R_1 & m+\frac{3}{4}n=-\frac{1}{4} \\ -\frac{1}{7}R_2 \to R_2 & n=1 \end{array}$$

Back substitution gives

$$\begin{aligned} m+\frac{3}{4}(1) &= -\frac{1}{4} \\ m &= -\frac{4}{4}=-1. \end{aligned}$$

The solution is $(m,n)=(-1,1)$.

$\square$

2.1.26

We wish to solve the system of three linear equations:

$$\begin{array}{lc} R_1 & 4x-y+3z=-2 \\ R_2 & 3x+5y-z=15 \\ R_3 & -2x+y+4z=14 \end{array}$$

First eliminate the x-terms from R_1 and R_2.

$$\begin{array}{rr} R_1 & 4x-y+3z=-2 \\ -3R_1+4R_2 \to R_2 & 23y-13z=66 \\ R_1+2R_3 \to R_3 & y+11z=26 \end{array}$$

Next, eliminate the y-term from R_3.

$$\begin{array}{rr} R_1 & 4x-y+3z=-2 \\ R_2 & 23y-13z=66 \\ R_2-23R_3 \to R_3 & -266z=-532 \end{array}$$

Now make the coefficient of the first term in each equation equal to 1.

$$\begin{array}{rcl} \frac{1}{4}R_1 \to R_1 & \qquad & x - \frac{1}{4}y + \frac{3}{4}z = -\frac{1}{2} \\ \frac{1}{23}R_2 \to R_2 & & y - \frac{13}{23} = \frac{66}{23} \\ -\frac{1}{266}R_3 \to R_3 & & z = 2 \end{array}$$

Complete the solution by back-substitution. Substitute 2 in for z in R_2 to find y.

$$\begin{aligned} y - \frac{13}{23}(2) &= \frac{66}{23} \\ y - \frac{26}{23} &= \frac{66}{23} \\ y &= \frac{92}{23} = 4 \end{aligned}$$

Finally, substitute 4 for y and 2 for z into R_1 to find x.

$$\begin{aligned} x - \frac{1}{4}(4) + \frac{3}{4}(2) &= -\frac{1}{2} \\ x + \frac{1}{2} &= -\frac{1}{2} \\ x &= -1 \end{aligned}$$

The solution is $(x, y, z) = (-1, 4, 2)$.

$\square$