# MATH 105: Assignment 2 Solutions

Each problem worth 4 points

**2.2.22**
We wish to solve the following system of two linear equations:

$$\begin{aligned} x + 2y &= 1 \\ 2x + 4y &= 3. \end{aligned}$$

Write down the augmented matrix for this system.

$$\begin{array}{c} R_1 \\ R_2 \end{array} \qquad \left(\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 4 & 3 \end{array}\right)$$

Use row operations to put the system in reduced form.

$$\begin{array}{r} R_1 \\ -2R_1 + R_2 \to R_2 \end{array} \qquad \left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 1 \end{array}\right)$$

The second row row represents equation $0x + 0y = 1$, which has no solution. Hence this system is inconsistent. $\square$

**2.2.34**
We want to solve the system of three linear equations:

$$\begin{aligned} 3 + 2y - z &= -16 \\ 6x - 4y + 3z &= 12 \\ 5x - 2y + 2z &= 4. \end{aligned}$$

Write down the augmented matrix for this system.

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \qquad \left(\begin{array}{ccc|c} 3 & 2 & -1 & 16 \\ 6 & -4 & 3 & 12 \\ 5 & -2 & 2 & 4 \end{array}\right)$$

Now use Gauss-Jordan elimination.

$$\begin{array}{r} R_1 \\ -2R_1 + R_2 \to R_2 \\ -5R_1 + 3R_3 \to R_3 \end{array} \qquad \left(\begin{array}{ccc|c} 3 & 2 & -1 & 16 \\ 0 & -8 & 5 & 44 \\ 0 & -16 & 11 & 92 \end{array}\right)$$

$$\begin{array}{r} R_2 + 4R_1 \to R_1 \\ R_2 \\ -2R_2 + R_3 \to R_3 \end{array} \qquad \left(\begin{array}{ccc|c} 12 & 0 & 1 & -20 \\ 0 & -8 & 5 & 44 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$$\begin{array}{r} -R_3 + R_1 \to R_1 \\ -5R_3 + R_2 \to R_2 \\ R_3 \end{array} \quad \left(\begin{array}{rrr|r} 12 & 0 & 0 & -24 \\ 0 & -8 & 0 & 24 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$$\begin{array}{r} 1/12R_1 \to R_1 \\ -1/8R_2 \to R_2 \\ R_3 \end{array} \quad \left(\begin{array}{rrr|r} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

The solution is $(x, y, z) = (-2, -3, 4)$. □

**2.2.46**

$$\begin{aligned} \text{Let } x &= \text{the number of chairs produced each week,} \\ y &= \text{the number of cabinets produced each week,} \\ z &= \text{the number of buffets produced each week.} \end{aligned}$$

Make a table to organize the information

| | Chair | Cabinet | Buffet | Totals |
|---|---|---|---|---|
| Cutting | .2 | .5 | .3 | 1950 |
| Assembly | .3 | .4 | .1 | 1490 |
| Finishing | .1 | .6 | .4 | 2160 |

The system to be solved is

$$\begin{aligned} .2x + .5y + .3z &= 1950 \\ .3x + .4y + .1z &= 1490 \\ .1x + .6y + .4z &= 2160. \end{aligned}$$

Write the augmented matrix of the system.

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \quad \left(\begin{array}{ccc|c} .2 & .5 & .3 & 1950 \\ .3 & .4 & .1 & 1490 \\ .1 & .6 & .4 & 2160 \end{array}\right)$$

Solve the system using Gauss-Jordan.

$$\begin{array}{r} 10R_1 \to R_1 \\ 10R_2 \to R_2 \\ 10R_3 \to R_3 \end{array} \quad \left(\begin{array}{ccc|c} 2 & 5 & 3 & 19,500 \\ 3 & 4 & 1 & 14,900 \\ 1 & 6 & 4 & 21,600 \end{array}\right)$$

$$\begin{array}{r} R_3 \to R_1 \\ R_2 \\ R_1 \to R_3 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 6 & 4 & 21,600 \\ 3 & 4 & 1 & 14,900 \\ 2 & 5 & 3 & 19,500 \end{array}\right)$$

$$\begin{array}{r} R_1 \\ -3R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 6 & 4 & 21,600 \\ 0 & -14 & -11 & -49,900 \\ 0 & -7 & -5 & -23,700 \end{array}\right)$$

$$\begin{array}{r} R_1 \\ -1/14R_2 \to R_2 \\ R_3 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 6 & 4 & 21,600 \\ 0 & 1 & 11/14 & 24,950/7 \\ 0 & -7 & -5 & -23,700 \end{array}\right)$$

$$\begin{array}{r} -6R_2 + R_1 \to R_1 \\ R_2 \\ 7R_2 + R_3 \to R_3 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 0 & -5/7 & 1500/7 \\ 0 & 1 & 11/14 & 24,950/7 \\ 0 & 0 & 1/2 & 1250 \end{array}\right)$$

$$\begin{array}{r} R_1 \\ R_2 \\ 2R_3 \to R_3 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 0 & -5/7 & 1500/7 \\ 0 & 1 & 11/14 & 24,950 \\ 0 & 0 & 1 & 2500 \end{array}\right)$$

$$\begin{array}{r} 5/7R_3 + R_1 \to R_1 \\ -11/14R_3 + R_2 \to R_2 \\ R_3 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2000 \\ 0 & 1 & 0 & 1600 \\ 0 & 0 & 1 & 2500 \end{array}\right)$$

The solution is $(x, y, z) = (2000, 1600, 2500)$. Therefore, 2000 chairs, 1600 cabinets, and 2500 buffets should be produced each week. □

**2.3.6**

Statement:

$$\begin{pmatrix} 2 & 4 & -1 \\ 3 & 7 & 5 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 4 & -1 \\ 3 & 7 & 5 \end{pmatrix}.$$

The statement is false since the matrices are different sizes. □

**2.3.22**

$$\begin{aligned} \begin{pmatrix} 1 & 5 \\ 2 & -3 \\ 3 & 7 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 8 & 5 \\ -1 & 9 \end{pmatrix} &= \begin{pmatrix} 1+2 & 5+3 \\ 2+8 & -3+5 \\ 3+(-1) & 7+9 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 8 \\ 10 & 2 \\ 2 & 16 \end{pmatrix} \end{aligned}$$

□

**2.3.28**

$$\begin{pmatrix}4 & 3\\ 1 & 2\end{pmatrix} - \begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix} + \begin{pmatrix}1 & 1\\ 1 & 4\end{pmatrix} = \begin{pmatrix}4 & 3\\ 1 & 2\end{pmatrix} + \begin{pmatrix}-1 & -1\\ -1 & 0\end{pmatrix} + \begin{pmatrix}1 & 1\\ 1 & 4\end{pmatrix}$$
$$= \begin{pmatrix}4+(-1)+1 & 3+(-1)+1\\ 1+(-1)+1 & 2+0+4\end{pmatrix}$$
$$= \begin{pmatrix}4 & 3\\ 1 & 6\end{pmatrix}$$

□

**2.4.20**

$$\begin{pmatrix}6 & 0 & -4\\ 1 & 2 & 5\\ 10 & -1 & 3\end{pmatrix}\begin{pmatrix}1\\ 2\\ 0\end{pmatrix} = \begin{pmatrix}6\cdot 1 + 0\cdot 2 + (-4)0\\ 1\cdot 1 + 2\cdot 2 + 5\cdot 0\\ 10\cdot 1 + (-1)2 + 3\cdot 0\end{pmatrix} = \begin{pmatrix}6\\ 5\\ 8\end{pmatrix}$$

□

**2.4.22**

$$\begin{pmatrix}-9 & 2 & 1\\ 3 & 0 & 0\end{pmatrix}\begin{pmatrix}2\\ -1\\ 4\end{pmatrix} = \begin{pmatrix}-9\cdot 2 + 2(-1) + 1\cdot 4\\ 3\cdot 2 + 0(-1) + 0\cdot 4\end{pmatrix} = \begin{pmatrix}-16\\ 6\end{pmatrix}$$

□

**2.4.28**

$$\begin{pmatrix}4 & 3\\ 1 & 2\\ 0 & -5\end{pmatrix}\left(\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}\begin{pmatrix}10\\ 0\end{pmatrix}\right) = \begin{pmatrix}4 & 3\\ 1 & 2\\ 0 & -5\end{pmatrix}\begin{pmatrix}20\\ 10\end{pmatrix} = \begin{pmatrix}110\\ 40\\ -50\end{pmatrix}$$

□

**2.4.30**

$$\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}\begin{pmatrix}4 & 3\\ 1 & 2\end{pmatrix} + \begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}\begin{pmatrix}7 & 0\\ -1 & 5\end{pmatrix} = \begin{pmatrix}6 & 2\\ 3 & 1\end{pmatrix} + \begin{pmatrix}16 & -10\\ 8 & -5\end{pmatrix} = \begin{pmatrix}22 & -8\\ 11 & -4\end{pmatrix}$$

□

**2.5.8**

$$\begin{pmatrix}-1 & 0 & 2\\ 3 & 1 & 0\\ 0 & 2 & -3\end{pmatrix}\begin{pmatrix}-1/5 & 4/15 & -2/15\\ 3/5 & 1/5 & 2/5\\ 2/5 & 2/15 & -1/15\end{pmatrix}$$
$$= \begin{pmatrix}1/5+0+4/5 & -4/15+0+4/15 & 2/15+0-2/15\\ -3/5+3/5+0 & 12/15+3/15+0 & -2/5+2/5+0\\ 0+6/5-6/5 & 0+2/5-2/5 & 0+4/5+1/5\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$\begin{pmatrix} -1/5 & 4/15 & -2/15 \\ 3/5 & 1/5 & 2/5 \\ 2/5 & 2/15 & -1/15 \end{pmatrix} \begin{pmatrix} -1 & 0 & 2 \\ 3 & 1 & 0 \\ 0 & 2 & -3 \end{pmatrix}$$
$$= \begin{pmatrix} 1/5+4/5+0 & 0+4/15-4/15 & -2/5+0+2/5 \\ -3/5+3/5+0 & 0+1/5+4/5 & 6/5+0-6/5 \\ -2/5+2/5+0 & 0+2/15-2/15 & 4/5+0+1/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Since both products are equal to the $3 \times 3$ identity matrix, the given matrices are inverses of eachother. □

**2.5.20**

Find the inverse of $A = \begin{pmatrix} 2 & 0 & 4 \\ 3 & 1 & 5 \\ -1 & 1 & -2 \end{pmatrix}$, if it exists.

$$(A|I) = \left(\begin{array}{ccc|ccc} 2 & 0 & 4 & 1 & 0 & 0 \\ 3 & 1 & 5 & 0 & 1 & 0 \\ -1 & 1 & -2 & 0 & 0 & 1 \end{array}\right)$$
$$\begin{array}{r} R_1 \\ 3R_1 + (-2)R_2 \to R_2 \\ R_1 + 2R_3 \to R_3 \end{array} \left(\begin{array}{ccc|ccc} 2 & 0 & 4 & 1 & 0 & 0 \\ 0 & -2 & 2 & 3 & -2 & 0 \\ 0 & 2 & 0 & 1 & 0 & 2 \end{array}\right)$$
$$\begin{array}{r} R_1 \\ R_2 \\ R_2 + R_3 \to R_3 \end{array} \left(\begin{array}{ccc|ccc} 2 & 0 & 4 & 1 & 0 & 0 \\ 0 & -2 & 2 & 3 & -2 & 0 \\ 0 & 0 & 2 & 4 & -2 & 2 \end{array}\right)$$
$$\begin{array}{r} -3R_3 + R_1 \to R_1 \\ -R_3 + R_2 \to R_2 \\ R_3 \end{array} \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & -7 & 4 & -4 \\ 0 & -2 & 0 & -1 & 0 & 1 \\ 0 & 0 & 2 & 4 & -2 & 2 \end{array}\right)$$
$$\begin{array}{r} 1/2R_1 \to R_1 \\ -1/2R_2 \to R_2 \\ 1/2R_3 \to R_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -7/2 & 2 & -2 \\ 0 & 1 & 0 & 1/2 & 0 & 1 \\ 0 & 0 & 1 & 2 & -1 & 1 \end{array}\right)$$

Thus, we find that

$$A^{-1} = \begin{pmatrix} -7/2 & 2 & -2 \\ 1/2 & 0 & 1 \\ 2 & -1 & 1 \end{pmatrix}.$$

□

**2.5.38**

$$\begin{aligned} 2x + 2y - 4z &= 12 \\ 2x + 6y + 0z &= 16 \\ -3x - 3y + 5z &= -20 \end{aligned}$$

This system may be written as the matrix equation

$$\begin{pmatrix} 2 & 2 & -4 \\ 2 & 6 & 0 \\ -3 & -3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 16 \\ -20 \end{pmatrix}$$

In Exercise 24, it was calculated that

$$\begin{pmatrix} 2 & 2 & -4 \\ 2 & 6 & 0 \\ -3 & -3 & 5 \end{pmatrix}^{-1} = \begin{pmatrix} -15/4 & -1/4 & -3 \\ 5/4 & 1/4 & 1 \\ -3/2 & 0 & -1 \end{pmatrix}.$$

$$\begin{aligned} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} -15/4 & -1/4 & -3 \\ 5/4 & 1/4 & 1 \\ -3/2 & 0 & -1 \end{pmatrix} \begin{pmatrix} 12 \\ 16 \\ -20 \end{pmatrix} \\ &= \begin{pmatrix} 11 \\ -1 \\ 2 \end{pmatrix} \end{aligned}$$

The solution is $(x, y, z) = (11, -1, 2)$. $\square$