MATH 105: Assignment 3 Solutions

Each question worth 4 points

Ch. 2 Review #6

We wish to solve the system of three linear equations:

$$\begin{array}{ll} R_1 & x - y + 0z = 3 \\ R_2 & 2x + 3y + z = 13 \\ R_3 & 3x + 0y - 2z = 21 \end{array}$$

First eliminate the x-terms from R_1 and R_2.

$$\begin{array}{rr} R_1 & x - y + 0z = 3 \\ 2R_1 - R_2 \to R_2 & -5y - z = -7 \\ 3R_1 - R_3 \to R_3 & -3y + 2z = -12 \end{array}$$

Next, eliminate the y-term from R_3.

$$\begin{array}{rr} R_1 & x - y + 0z = 3 \\ R_2 & -5y - z = -7 \\ 3R_2 - 5R_3 \to R_3 & -13z = 39 \end{array}$$

Finally, make the coefficient of the first term in each equation equal to 1.

$$\begin{array}{rr} R_1 \to R_1 & x - y + 0z = 3 \\ -1/5R_2 \to R_2 & y + 1/5z = 7/5 \\ -1/13R_3 \to R_3 & z = -3 \end{array}$$

Complete the solution by back-substitution.

$$\begin{aligned} z &= -3 \\ y &= 7/5 - 1/5(-3) = 2 \\ x &= 3 + 2 = 5 \end{aligned}$$

The solution is $(x, y, z) = (5, 2, -3)$. □

Ch. 2 Review #10

We wish to solve the system of three linear equations:

$$\begin{aligned} x + 0y - 2z &= 5 \\ 3x + 2y + 0z &= 8 \\ -x + 0y + 2z &= 10 \end{aligned}$$

Write the system as an augmented matrix and use Gauss-Jordan.

$$\begin{array}{r} R_1 \\ R_2 \\ R_3 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 3 & 2 & 0 & 8 \\ -1 & 0 & 2 & 10 \end{array}\right)$$

$$\begin{array}{r} R_1 \\ -3R_1 + R_2 \to R_2 \\ R_1 + R_3 \to R_3 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 2 & 6 & -7 \\ 0 & 0 & 0 & 15 \end{array}\right)$$

The last row represents the equation $0x + 0y + 0z = 15$, which has no solution. Hence this system is inconsistent. $\square$

Ch. 2 Review #34

Find the inverse of $A = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & -2 & 0 \end{pmatrix}$, if it exists.

The augmented matrix is

$$(A|I) = \left(\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} R_1 \\ -2R_2 + R_1 \to R_2 \\ -2R_3 + R_2 \to R_3 \end{array} \left(\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -2 & 1 & -2 & 0 \\ 0 & 3 & 0 & 1 & 0 & -2 \end{array}\right)$$

$$\begin{array}{r} -1R_2 + R_1 \to R_1 \\ R_2 \\ 3R_1 + R_3 \to R_3 \end{array} \left(\begin{array}{ccc|ccc} 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & -1 & -2 & 1 & -2 & 0 \\ 0 & 0 & -6 & 4 & -6 & -2 \end{array}\right)$$

$$\begin{array}{r} 3R_1 + R_3 \to R_1 \\ -3R_2 + R_3 \to R_2 \\ R_3 \end{array} \left(\begin{array}{ccc|ccc} 6 & 0 & 0 & 4 & 0 & -2 \\ 0 & 3 & 0 & 1 & 0 & -2 \\ 0 & 0 & -6 & 4 & -6 & -2 \end{array}\right)$$

$$\begin{array}{r} \frac{1}{6}R_1 \to R_1 \\ \frac{1}{3}R_2 \to R_2 \\ -\frac{1}{6}R_3 \to R_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{3} & 0 & -\frac{2}{3} \\ 0 & 0 & 1 & -\frac{2}{3} & 0 & \frac{1}{3} \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} \frac{2}{3} & 0 & -\frac{1}{3} \\ \frac{1}{3} & 0 & -\frac{2}{3} \\ -\frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

Ch. 2 Review #54

(a) $\begin{pmatrix} 3170 \\ 2360 \\ 1800 \end{pmatrix}$

(b) $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(c) $\begin{pmatrix} 10 & 5 & 8 \\ 12 & 0 & 4 \\ 0 & 10 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3170 \\ 2360 \\ 1800 \end{pmatrix}$

(d) If possible, this is a good matrix inverse to find using a calculator or computer.

$$\begin{aligned}
\begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 10 & 5 & 8 \\ 12 & 0 & 4 \\ 0 & 10 & 5 \end{pmatrix}^{-1} \begin{pmatrix} 3170 \\ 2360 \\ 1800 \end{pmatrix} \\
&= \begin{pmatrix} -.154 & .212 & .0769 \\ -.231 & .192 & .2154 \\ .462 & -.385 & -.231 \end{pmatrix} \begin{pmatrix} 3170 \\ 2360 \\ 1800 \end{pmatrix} \\
&= \begin{pmatrix} 150 \\ 110 \\ 140 \end{pmatrix}
\end{aligned}$$

□

7.1.34

$X \cup Y$, the union of X and Y, is the set of all elements belonging to X or Y or both. Thus,

$$X \cup Y = \{2, 3, 4, 5, 7, 9\}.$$

□

7.1.38

$X' \cap Z$, the intersection of X' and Z, is the set of elements that are in Z but are not in X. Thus,

$$X' \cap Z = \{7, 9\}.$$

□

7.1.54

$B = \{a, b, c, \{d\}, \{e, f\}\}$

(a) $a \in B$ is true.

(b) $\{a, c, d\} \subset B$ is false. $(d \notin B)$

(c) $\{d\} \in B$ is true.

(d) $\{d\} \subseteq B$ is false. $(\{d\} \in B)$

(e) $\{e, f\} \in B$ is true.

(f) $\{a, \{e, f\}\} \subset B$ is true.

(g) $\{e, f\} \subset B$ is false. $(\{e, f\} \in B)$

□

7.2.22

$$\begin{aligned}
n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\
35 &= 13 + n(B) - 5 \\
35 &= 8 + n(B) \\
n(B) &= 27
\end{aligned}$$

□

7.2.26

$n(A') = 28$
$n(B) = 25$
$n(A' \cup B') = 45$
$n(A \cap B) = 12$

$n(B) = 25$ and $n(A \cap B) = 12$, so $n(B \cap A') = 13$. Since $n(A') = 28$, of which 13 elements are accounted for, 15 elements are in $A' \cap B'$.

$$\begin{aligned} n(A' \cup B') &= n(A') + n(B') - n(A' \cap B') \\ 45 &= 28 + n(B') - 15 \\ 45 &= 13 + n(B') \\ 32 &= n(B') \end{aligned}$$

15 elements are in $A' \cap B'$, so the rest are in $A \cap B'$, and $n(A \cap B') = 17$. □

7.2.36

Let m be the set of those who use a microwave oven, E be the set of those who use an electric range, and G be the set of those who use a gas range. We are given the following information

$$\begin{aligned} &n(U) = 140 \\ &n(M) = 58 \\ &n(E) = 63 \\ &n(G) = 58 \\ &n(M \cap E) = 19 \\ &n(M \cap G) = 17 \\ &n(G \cap E) = 4 \\ &n(M \cap G \cap E) = 1 \\ &n(M' \cap G' \cap E') = 2 \end{aligned}$$

Since $n(M \cap G \cap E) = 1$, there is 1 element in the region where the three sets overlap. Since $n(M \cap E) = 19$, there are $19 - 1 = 18$ elements in $M \cap E$ but not in $M \cap G \cap E$. Since $n(M \cap G) = 17$, there are $17 - 1 = 16$ elements in $M \cap G$ but not in $M \cap G \cap E$. Since $n(G \cap E) = 4$, ther eare $4 - 1 = 3$ elements in $M \cap E$ but not in $M \cap G \cap E$. Now consider $n(M) = 58$. So far we have $16 + 1 + 18 = 35$ in m; there must be another $58 - 35 = 23$ in M not yet counted. Similarly, $n(E) = 63$; we have $18 + 1 + 3 = 22$ counted so far. There must be $63 - 22 = 41$ more in E not yet counted. Also, $n(G) = 58$; we have $16 + 1 + 3 = 20$ counted so far. There must be $58 - 20 - 38$ more in G not yet counted. Lastly, $n(M' \cap G' \cap E') = 2$ indicates that there are 2 elements outside all of the three sets.
Note that the numbers in the Venn diagram add up to 142 even though $n(U) = 140$. Jeff has made some error, and he should definitely be fired. □

7.3.14

$S = \{Y_1, Y_2, Y_3, W_1, W_2, W_3, W_4, B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8\}$

(a) Let E be the event "drawing a yellow marble". $E = \{Y_1, Y_2, Y_3, Y_4\}$.

(b) Let F be the event "drawing a white marble". $F = \{W_1, W_2, W_3, W_4\}$.

(c) Let G be the event "drawing a blue marble". $B = \{B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8\}$.

(d) Let H be the event "drawing a black marble". $H = \{\} = \emptyset$.

□

7.3.32

Let B be the event "drawing a black 7" and let R be the event "drawing a red 8". Then $B \cup R$ represents the event "drawing a black 7 or a red 8". Since the events are mutually exclusive, we have

$$\begin{aligned} P(B \cup R) &= P(B) + P(R) \\ &= \frac{2}{52} + \frac{2}{52} \\ &= \frac{4}{52} = \frac{1}{13}. \end{aligned}$$

□

7.3.40

There are 8 marbles which are orange or yellow.

$$P(\text{ orange or yellow }) = \frac{8}{18} = \frac{4}{9}$$

□