# MATH 105: Assignment 4 Solutions

Each problem worth 4 points

### 7.4.16

**(a)** The cards less than 4 include 2's, 3's, and aces. There are a total of 12 of these cards in a deck of 52, so

$$P(\text{ ace or 2 or 3 }) = \frac{12}{52} = \frac{3}{13}.$$

**(b)** There are 13 diamonds plus three 7's in other suits, so

$$P(\text{ diamond or 7 }) = \frac{16}{52} = \frac{4}{13}.$$

**(c)** There are 26 black cards plus 2 red aces, so

$$P(\text{ black or ace }) = \frac{28}{52} = \frac{7}{13}.$$

**(d)** There are 13 hearts and 3 jacks which are not hearts, so

$$P(\text{ heart or jack }) = \frac{16}{52} = \frac{4}{13}.$$

**(e)** There are 26 red cards plus 6 black face cards, so

$$P(\text{ red or face card }) = \frac{32}{52} = \frac{8}{13}.$$

$\square$

### 7.4.18

**(a)** There are 3 uncles plus 2 cousins out of 10, so

$$P(\text{uncle or cousin}) = \frac{5}{1} = \frac{1}{2}.$$

**(b)** There are 3 uncles, 2 brothers, and 2 cousins, for a total of 7 out of 10, so

$$P(\text{male or cousin}) = \frac{7}{10}.$$

**(c)** There are 2 aunts, 2 cousins, and 1 mother, for a total of 5 out of 10, so

$$P(\text{female or cousin}) = \frac{5}{10} = \frac{1}{2}.$$

$\square$

### 7.4.54

**(a)**

$$\begin{aligned} P(\$500 \text{ or more}) &= 1 - P(\text{less than } \$500) \\ &= 1 - (.31 + .18) \\ &= 1 - .49 \\ &= .51 \end{aligned}$$

**(b)**

$$\begin{aligned} P(\text{less than } \$1000) &= .31 + .18 + .18 \\ &= .67 \end{aligned}$$

**(c)**

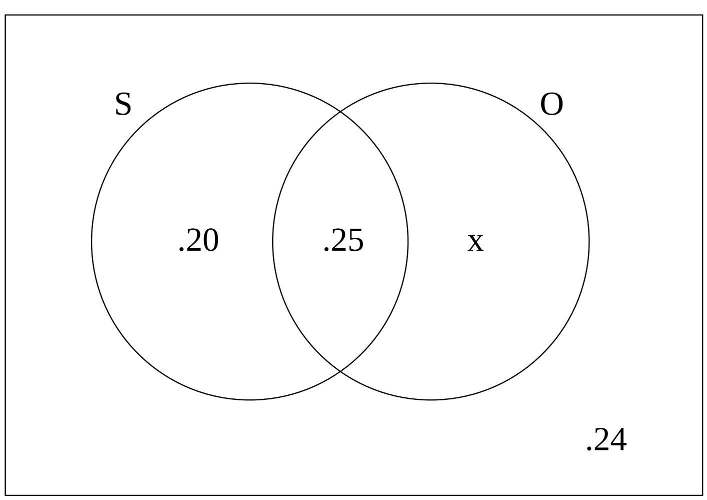
$$\begin{aligned} P(\$500 \text{ to } \$2999) &= .18 + .13 + .08 \\ &= .39 \end{aligned}$$

**(d)**

$$\begin{aligned} P(\$3000 \text{ or more}) &= .05 + .06 + .01 \\ &= .12 \end{aligned}$$

□

**7.4.58** Let $S$ be the event "the person is short," and let $O$ be the event "the person is overweight."




From the Venn diagram,

$$\begin{aligned} .20 + .25 + x + .24 &= 1 \\ .69 + x &= 1 \\ x &= .31. \end{aligned}$$

The probability that a person is

**(a)** overweight is $.25 + .31 = .56$;

**(b)** short, but not overweight is $.20$;

**(c)** tall (not short) and overweight is $.31$.

$\square$

**7.4.62** We have a probability distribution with three outcomes, as in the box on page 351.

**(a)** $P($ no more than 4 good toes $) = .77 + .13 = .90$

**(b)** $P($ 5 toes $) = .13 + .10 = .23$

$\square$

The next two quesions refer to the experiment: two cards are drawn from an ordinary deck without replacement.

**7.5.8** Let $E$ be the event "the second is black." Let $F$ be the event "the first is a spade." The probability that the second is black, given that the first is a space is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\mathrm{n}(E \cap F)}{\mathrm{n}(F)}.$$

We will count the sets $E \cap F$ and $F$. $E \cap F$ is the event "the first is a spade **and** the second is black." There are 13 ways to choose the first spade, and then there are 25 black cards remaining for our second choice. Hence $\mathrm{n}(E \cap F) = 13 \cdot 25$. To count $F$, there are 13 ways to choose the first spade, then there are 51 cards left for the second choice. Hence $\mathrm{n}(F) = 13 \cdot 51$. We conclude that

$$P(E|F) = \frac{\mathrm{n}(E \cap F)}{\mathrm{n}(F)} = \frac{13 \cdot 25}{13 \cdot 51} = \frac{25}{51}.$$

**7.5.14** First, let us count the sample space $S =$ "pairs of cards taken from a deck without replacement." There are 52 ways to choose the first card, then 51 cards remaining for the second choice. Thus, $\mathrm{n}(S) = 52 \cdot 51$. Let $E$ be the event "two hearts are drawn." To count these, notice that there are 13 ways to choose the first heart, then there are 12 hearts remaining for the second choice. Thus, $\mathrm{n}(E) = 13 \cdot 12$. Then the probability of $E$ is

$$P(E) = \frac{\mathrm{n}(E)}{\mathrm{n}(S)} = \frac{13 \cdot 12}{52 \cdot 51} = \frac{3}{51}.$$

**7.5.36** Let $W$ be the event "withdraw cash from ATM" and let $C$ be the event "check account balance at ATM". We are given $P(C) = .23$, $P(W) = .92$, and

$P(C \cup W) = .96$. Then we have

$$\begin{aligned} P(C \cup W) &= P(C) + P(W) - P(C \cap W) \\ .96 &= .32 + .92 - P(C \cap W) \\ -.28 &= -P(C \cap W) \\ P(C \cap W) &= .28. \end{aligned}$$

Then the probability that someone withdraws cash from the ATM, given that they check their account balance is

$$\begin{aligned} P(W|C) &= \frac{P(C \cap W)}{P(C)} \\ &= \frac{.28}{.32} \\ &\approx .875. \end{aligned}$$

$\square$

**7.5.42** Every edge in the tree gets labelled with 1/2, and so each branch has an equal probability of 1/8.

$$\begin{aligned} P(\text{ 3 girls} \,|\, \text{3rd is a girl }) &= \frac{P(\text{ 3 girls and 3rd is a girl })}{P(\text{ 3rd is a girl })} \\ &= \frac{1/8}{1/8 + 1/8 + 1/8 + 1/8} \\ &= \frac{1}{4} \end{aligned}$$

$\square$