MATH 105: Assignment 6 Solutions

Each problem worth 4 points

8.2.10 We wish to choose a hand of 6 clubs, from a total of 13 clubs. The number of different ways we can do this is

$$\binom{13}{6} = \frac{13!}{6!7!} = 1716.$$

$\square$

8.2.14 Find the number of ways of choosing 2 letters from the set $\{L, M, N, P\}$ if

(a) order is important and repetition is allowed. There are 4 ways to choose the first letter, then 4 ways to choose the second, for a total of $4 \cdot 4 = 16$ choices.

(b) order is important, and repetition is not allowed. There are 4 ways to choose the first letter, then 3 ways to choose the second, for a total of $P(4, 2) = 4 \cdot 3 = 12$ choices.

$\square$

8.2.22

(a) There are $\binom{25}{3} = 2,300$ ways to choose a sample of 3 apples.

(b) There are $\binom{5}{3} = 10$ ways to choose 3 rotten apples, and then $\binom{20}{0} = 1$ way to choose 0 good apples, for a total of $\binom{5}{3}\binom{20}{0} = 10 \cdot 1 = 10$ choices.

(c) There are $\binom{5}{1} = 5$ ways to choose the rotten apple, then $\binom{20}{2} = 190$ ways to choose the two good apples, for a total of $\binom{5}{1}\binom{20}{2} = 5 \cdot 190 = 950$ choices.

$\square$

8.2.30 A group of 7 workers selects a delegation of 2.

(a) There are $\binom{7}{2} = 21$ ways to do this.

(b) If we fix one person, say ‘Mary’, to be in the delegation, then there are $\binom{6}{1} = 6$ ways to select someone to go with her.

(c) If the delegation contains 'at least 1 woman', then it contains either 1 or 2 women. In the first case, there are $\binom{2}{1}\binom{5}{1}$ possibilities, and in the second case there are $\binom{2}{2}\binom{5}{0}$ possibilities, for a total of

$$\binom{2}{1}\binom{5}{1} + \binom{2}{2}\binom{5}{0} = 2 \cdot 5 + 1 \cdot 1 = 11$$

possibilities.

8.2.34

(a) There are 11 plants, so there are $\binom{11}{4} = 330$ ways to select 4 of them.

(b) Now we must include exactly 2 wheat plants. There are $\binom{6}{2} = 15$ ways to select the wheat plants, then there are $\binom{5}{2} = 10$ ways to select the other 2 plants from the remaining 5, for a total of $15 \cdot 10 = 150$ ways.

$\square$

8.3.4 "More red than yellow" means 2 or 3 reds. Since there are 6 reds and 4 yellows total, there are $\binom{6}{2}\binom{4}{1}$ ways to choose 2 reds, and $\binom{6}{3}\binom{4}{0}$ ways to choose 3 reds. Since there are $\binom{10}{3}$ ways to choose three apples, we have

$$P(\text{"more red than yellow"}) = \frac{\binom{6}{2}\binom{4}{1} + \binom{6}{3}\binom{4}{0}}{\binom{10}{3}} = \frac{15 \cdot 4 + 20 \cdot 1}{120} = \frac{80}{120} = \frac{2}{3}.$$

8.3.10 There are 12 face cards in a standard deck of 52, so there are $\binom{12}{2} = 66$ ways to choose a hand with two face cards. There are $\binom{52}{2} = 1326$ possible hands, so the probability of getting only face cards is

$$P(\text{only face cards}) = \frac{\binom{12}{2}}{\binom{52}{2}} = \frac{66}{1326} \approx .0498.$$

$\square$

8.3.16 There are $26 - 3 = 23$ letters that are not 'x', 'y', or 'z', and there are 23^5 words that can be formed using these letters. Since there are a total of 26^5 5-letter words, the probability of getting a word with no 'x', 'y', or 'z' is

$$P(\text{no 'x', 'y', or 'z'}) = \frac{23^5}{26^5} = \frac{6,436,343}{11,881,376} \approx .5417.$$

$\square$

8.3.42 A straight could start with an Ace, 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10 as the low card, giving 10 choices. Then how many ways could we choose the suits?

Since there are 4 possible suits, there are 4^5 ways that we could choose the suits, but 4 of these will be flushes (all clubs, all spades, all diamonds, all hearts), and we do not wish to count these, so there are a total of $10 \cdot (4^5 - 4) = 10,200$ straights. Since there are $\binom{52}{5}$ total poker hands, the probability of getting a straight is

$$\frac{10 \cdot (4^5 - 4)}{\binom{52}{5}} = \frac{10,200}{2,598,960} \approx .00392.$$

8.3.50 There are a total of 21, so the total number of 6-book selections is $\binom{21}{6} = 54,264$.

(a) There are $\binom{9}{3}$ ways to choose the 3 Hughes books, and $\binom{7}{3}$ ways to choose the 3 Morrison books, so the probability is

$$\frac{\binom{9}{3}\binom{7}{3}}{\binom{21}{6}} = \frac{85 \cdot 35}{54,264} \approx .054.$$

(b) There are $\binom{5}{4}$ ways to choose the 4 Baldwin books, then there are $\binom{16}{2}$ ways to choose the remaining books from the other two authors, so the probability is

$$\frac{\binom{5}{4}\binom{16}{2}}{\binom{21}{6}} = \frac{5 \cdot 120}{54,264} \approx .011.$$

(d) A selection with at least 4 Hughes books may contain 4, 5, or 6 Hughes books, with any remaining books by the other two authors. Therefore, the probability is

$$\begin{aligned}\frac{\binom{9}{4}\binom{12}{2} + \binom{9}{5}\binom{12}{1} + \binom{9}{6}\binom{12}{0}}{\binom{21}{6}} &= \frac{126 \cdot 66 + 126 \cdot 12 + 84 \cdot 1}{54,264} \\ &= \frac{9912}{54,264} \approx .183.\end{aligned}$$

$\square$

8.3.54 There are $\binom{270}{4}$ ways to choose the sayings for a Barbie doll. Now, if the saying "Math class is tough" must be included, there are $\binom{1}{1}$ ways to choose this saying, then $\binom{269}{3}$ ways to choose the other three sayings on the doll. So the probability is

$$P(\text{doll says "Math class is tough"}) = \frac{\binom{1}{1}\binom{269}{3}}{\binom{270}{4}} \approx .0148.$$

So there is a 1.48% chance of getting a Barbie that says "Math class is tough". The spokeswoman from Mattel was lying. $\square$