MATH 105: Assignment 7 Solutions

Each problem worth 4 points

Chapter 7 Review Exercise 44. Let S be the sample space of all 52 cards, let J be the event "the card is a Jack", and let F be the event "the card is a face card". Then $n(F) = 12$ since there are 12 face cards, and $n(J \cap F) = 4$, since there are 4 cards which are both a Jack and a face card ($J \cap F = J$ because $J \subseteq F$). So the conditional probability $P(J|F)$ is given by

$$P(J|F) = \frac{P(J \cap F)}{P(F)} = \frac{n(J \cap F)/n(S)}{n(F)/n(S)} = \frac{n(J \cap F)}{n(F)} = \frac{4}{12} = \frac{1}{3}.$$

□

Chapter 7 Review Exercise 60. Let S be the sample space of all 36 possible rolls. Let E be the event "the sum of the two dice is 12" and let F be the event "the sum of the two dice is greater than 10". Then $n(F) = 3$, since there are two ways to roll 11 and one way to roll 12, and $n(E \cap F) = 1$, since there is only one way for the sum to be both "12" and "greater than 10" (note that $E \cap F = E$ since $E \subseteq F$). Hence the conditional probability $P(E|F)$ is

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{n(E \cap F)/n(S)}{n(F)/n(S)} = \frac{n(E \cap F)}{n(F)} = \frac{1}{3}.$$

□

Chapter 7 Review Exercise 72. Let E be the event "a customer buys a printer", and let F be the event "a customer buys a copier".

(a) The event "a customer buys neither machine" is given by $E' \cap F'$ ($= (E \cup F)'$ by de Morgan's law), since $E' \cap F'$ means that the customer does not buy a printer **and** the customer does not buy a copier.

(b) The event "a customer buys at least one of the machines" is given by $E \cup F$, since $E \cup F$ means the customer buys a printer **and/or** the customer buys a copier. Note that this is the complement to the event in part (a).

□

Chapter 7 Review Exercise 78. Let F be the event "the voter is female" and let R be the event "the voter is Republican". We are given that $P(R|F) =$

.27, $P(R|F') = .36$, $P(F) = .51$ and $P(F') = .49$. Using Bayes' Theorem we get

$$\begin{aligned} P(F|R) &= \frac{P(F)\cdot P(R|F)}{P(F)\cdot P(R|F) + P(F')\cdot P(R|F')} \\ &= \frac{.51(.27)}{.51(.27) + .49(.36)} \\ &= \frac{.1377}{.3141} \approx .44 \end{aligned}$$

hence 44% of Republicans are women.

Chapter 8 Review Exercise 4. We have a bag of 12 oranges in which 2 of them are rotten and 10 of them are not rotten. We reach in and grab 3 oranges.

(a) The number of ways we can get 1 rotten orange and 2 good oranges is $\binom{2}{1}\binom{10}{2} = 2 \cdot 45 = 90$.

(d) In how many ways can we get at least 2 rotten oranges. There are $\binom{2}{0}\binom{10}{3} = 120$ ways to get 0 rotten oranges, $\binom{2}{1}\binom{10}{2} = 90$ ways to get 1 rotten orange, and $\binom{2}{2}\binom{10}{1} = 10$ ways to get 2 rotten oranges, for a total of 220 ways.

Alternatively, we can see that **any** choice of 3 oranges from the bag will contain at most 2 rotten oranges, since there are only 2 rotten oranges in there. So every choice of 3 oranges is okay, and there are $\binom{12}{3} = 220$ choices.

□

Chapter 8 Review Exercise 8. There are 8 items in column A and 6 items in column B.

(a) How many ways are there to select 5 items from the menu, if 3 of them must be from column A and 2 of them must be from column B? Answer: $\binom{8}{3}\binom{6}{2} = 56 \cdot 15 = 840$.

(b) How many ways can we select up to 3 items from column A and up to 2 items from column B, if the total number of items must be greater than zero? Answer: There are $\binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \binom{8}{3}$ ways to choose up to 3 items from column A, and $\binom{6}{0} + \binom{6}{1} + \binom{6}{2}$ ways to choose up to 2 items from column B. Since our choices in column A are independent of our choices in column B, there are

$$\begin{aligned} \left(\binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \binom{8}{3}\right)\left(\binom{6}{0} + \binom{6}{1} + \binom{6}{2}\right) &= 93 \cdot 22 \\ &= 2046 \end{aligned}$$

possible choices. But 1 of these choices is to select zero items from both columns. Subtracting off this choice, we get $2046-1 = 2045$ possibilities.

□

Chapter 8 Review Exercise 14. A basket contains 11 balls: 4 black, 2 blue, and 5 green. We reach in and grab 3. What is the probability that we get 2 black balls and 1 green ball? Well, there are $\binom{4}{2}\binom{2}{0}\binom{5}{1}$ ways to do this, and there are $\binom{11}{3}$ total possibilities. So the probability is

$$\frac{\binom{4}{2}\binom{2}{0}\binom{5}{1}}{\binom{11}{3}} = \frac{6 \cdot 1 \cdot 5}{165} = \frac{30}{165} \approx .182.$$

□

Chapter 8 Review Exercise 20. A family of 6 children can be represented by a string of 6 b's and g's, for example *bggbgb*. There are $2^6 = 64$ different possibilities. What is the probability that a family with 6 children has at least 4 girls? Such a family could have 4, 5, or 6 girls. To determine a family, we only need to choose the places to put the girls (the example above puts the girls in places 2, 3, and 5). Hence there are $\binom{6}{4}$ 4-girl families, $\binom{6}{5}$ 5-girl families, and $\binom{6}{6}$ 6-girl families. So the probability is

$$\frac{\binom{6}{4}+\binom{6}{5}+\binom{6}{6}}{64} = \frac{15+6+1}{64} = .34375.$$

□

Chapter 8 Review Exercise 24. A hand of 2 cards is dealt from a standard deck of 52. There are $\binom{52}{2} = 1326$ possible hands. What is the probability that we get at least one spade? Well, there are $\binom{39}{2} = 741$ ways to choose a hand that has **no** spades, hence the probability of getting at least one spade is

$$1 - \frac{\binom{39}{2}}{\binom{52}{2}} = 1 - \frac{741}{1326} = \frac{585}{1326} \approx .441.$$

□