MATH 105: Assignment 8 Solutions

Each problem worth 4 points

8.4.22. The properties of a binomial experiment are listed on page 424. Clearly the first two properties are satisfied by this experiment (the experiment is repeated twice, and on each trial there are exactly two possible outcomes). However, the third property of a binomial experiment may not be satisfied. Namely, it is not clear that the two crib deaths are independent of eachother. There are many possible ways in which the deaths could be related, for instance by genetic predisposition (or by murder). □

8.4.40. Let "success" indicate producing a defective item. Then this is a binomial experiment with $n = 75$ and $p = .05$.

(a) The probability of producing exactly 5 defective items is

$$\begin{aligned} P(\text{ exactly 5 defective }) &= \binom{75}{5}(.05)^5(.95)^{70} \\ &\approx .149. \end{aligned}$$

(b) The probability of producing zero defective items is

$$\begin{aligned} P(\text{ zero defective }) &= \binom{75}{0}(.05)^0(.95)^{75} \\ &\approx .021. \end{aligned}$$

(c) The probability of producing at least 1 defective item is

$$\begin{aligned} P(\geq 1 \text{ defective }) &= 1 - P(\text{ zero defective }) \\ &\approx 1 - .021 \\ &= .979. \end{aligned}$$

□

8.4.64. Let "success" be getting the correct answer. There are $n = 10$ questions and on each question we have a $p = \frac{1}{5}$ chance of success. Assume that getting the correct answers on any two questions is independent. Then the probability of getting less than 8 correct answers is

$$\begin{aligned} P(< 8 \text{ correct }) &= 1 - P(\geq 8 \text{ correct }) \\ &= 1 - \binom{10}{8}(.2)^8(.8)^2 - \binom{10}{9}(.2)^9(.8)^1 - \binom{10}{10}(.2)^{10}(.8)^0 \\ &= 1 - .000074 - .000004 - .0000001 \\ &= .999922. \end{aligned}$$

$\square$

8.5.4. Two balls are drawn from a bag containing 4 white and 2 black balls. Let x equal the number of black balls drawn. There are three possibilities: $x = 0$, 1, or 2. The corresponding probabilities are

$$P(x=0) = \frac{\binom{2}{0}\binom{4}{2}}{\binom{6}{2}} = \frac{6}{15},$$
$$P(x=1) = \frac{\binom{2}{1}\binom{4}{1}}{\binom{6}{2}} = \frac{8}{15},$$
$$P(x=2) = \frac{\binom{2}{2}\binom{4}{0}}{\binom{6}{2}} = \frac{1}{15}.$$

This gives a probability distribution

x	0	1	2
$P(x)$	$\frac{6}{15}$	$\frac{8}{15}$	$\frac{1}{15}$

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$\square$

8.5.14. This histogram corresponds to a probability distribution

x	2	4	6	8	10
$P(x)$	.2	.3	.2	.1	.2

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Hence, the expected value of x is

$$E(x) = 2(.2) + 4(.3) + 6(.2) + 8(.1) + 10(.2) = 5.6.$$

$\square$

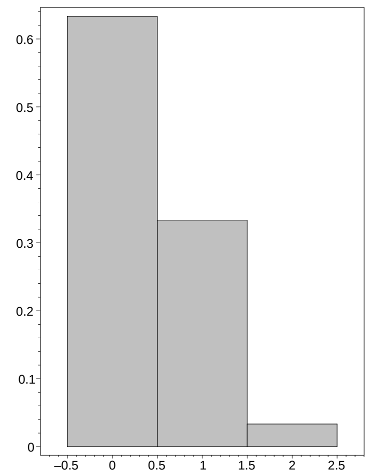
8.5.20. A barrel contains 5 rotten apples and 20 good ones. We reach in and grab 2. Let x equal the number of rotten apples we get.

(a) There are three possibilities for x: $x = 0$, 1, or 2. We calculate the distribution

x	0	1	2
$P(x)$	$\frac{\binom{5}{0}\binom{20}{2}}{\binom{25}{2}}$ $\frac{190}{300}$	$\frac{\binom{5}{1}\binom{20}{1}}{\binom{25}{2}}$ $\frac{100}{300}$	$\frac{\binom{5}{2}\binom{20}{0}}{\binom{25}{2}}$ $\frac{10}{300}$

,

and plot the histogram



(b) Therefore, the expected value of x is

$$E(x) = 0\left(\frac{190}{300}\right) + 1\left(\frac{100}{300}\right) + 2\left(\frac{10}{300}\right) = \frac{120}{300} = .4.$$

On average, we will get .4 rotten apples in a sample of 2. □

8.5.40. Suppose that there is 83% chance that a mailed letter will arrive the next day. We do this 10 times. Then this is a binomial experiment with $n = 10$ and $p = .83$.

(a) Let x equal the number of letters that arrive the next day. The distribution of x is

x	$P(x)$
0	$\binom{10}{0}(.83)^0(.17)^{10} \approx .0000$
1	$\binom{10}{1}(.83)^1(.17)^9 \approx .0000$
2	$\binom{10}{2}(.83)^2(.17)^8 \approx .0000$
3	$\binom{10}{3}(.83)^3(.17)^7 \approx .0003$
4	$\binom{10}{4}(.83)^4(.17)^6 \approx .0024$
5	$\binom{10}{5}(.83)^5(.17)^5 \approx .0141$
6	$\binom{10}{6}(.83)^6(.17)^4 \approx .0573$
7	$\binom{10}{7}(.83)^7(.17)^3 \approx .1600$
8	$\binom{10}{8}(.83)^8(.17)^2 \approx .2929$
9	$\binom{10}{9}(.83)^9(.17)^1 \approx .3178$
10	$\binom{10}{10}(.83)^{10}(.17)^0 \approx .1552$

(b)

$$\begin{aligned} P(x \le 4) &= P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) \\ &\approx .0000 + .0000 + .0000 + .0003 + .0024 \\ &\approx .0027 = .27\% \end{aligned}$$

(c) When Mr. Statistics performed this experiment he had 4 letters delivered the next day. According to the U.S. Postal Service figures, the chance of this happening is less than .27%. So, either Mr. Statistics is very, very unlucky, or the U.S. Postal service figure of 83% next day delivery is somewhat exaggerated.

(d) The expected number of letters delivered the next day is

$$\begin{aligned} E(x) &\approx 0(.0000) + 1(.0000) + 2(.0000) + \cdots + 9(.3178) + 10(.1552) \\ &= 8.3. \end{aligned}$$

Or, we could refer to page 439, which states that the expected value of a binomial random variable is np, in this case $np = 10(.83) = 8.3$.

$\square$