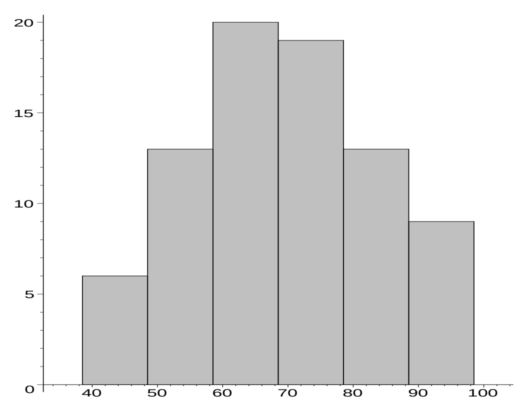
MATH 105: Assignment 9 Solutions

Each problem worth 4 points

9.1.4. Here is the frequency distribution and histogram. You'll have to imagine the frequency polygon in your mind.

Interval	Frequency
39-48	6
49-58	13
59-68	20
69-78	19
79-88	13
89-98	9



□

9.1.20. To find the median for the distribution .6, .4, .9, 1.2, .3, 4.1, 2.2, .4, .7, .1., we first arrange the data in order:

$$.1, .3, .4, .4, .6, .7, .9, 1.2, 2.2, 4.1.$$

Since there are an even number of data points, the median is the average of the two middle numbers

$$\text{median } = \tilde{x} = \frac{.6 + .7}{2} = .65.$$

□

9.1.38a. Find the mean for the grouped data.

$$
\begin{aligned}
\bar{x} &= \frac{\sum fx}{n} \\
&= \frac{(2500)(2037) + (7500)(4870) + \cdots + (87,000)(9740)}{2037 + 4870 + \cdots + 9740} \\
&= \frac{3,185,002,500}{75,973} \approx 41,923
\end{aligned}
$$

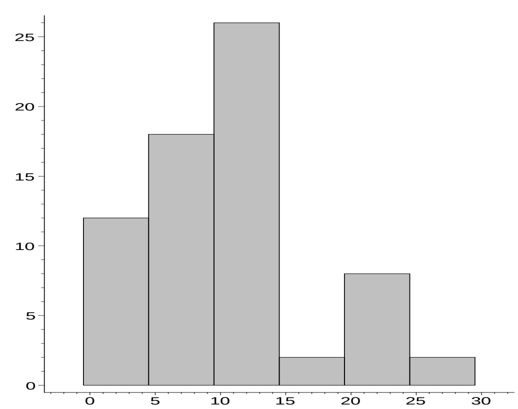
The estimated mean household income for full-time white Americans in 2000 is \$41,923. □

9.1.44.

(a)

Interval	Frequency
0-4	6
5-9	9
10-14	13
15-19	1
20-24	4
25-29	1

(b)



(c) For the original data, $\sum x = 355$ and $n = 34$, so

$$\bar{x} = \frac{\sum x}{n} = \frac{355}{34} \approx 10.44.$$

(d)

Interval	Midpoint x	Frequency f	Product fx
0-4	2	6	12
5-9	7	9	63
10-14	12	13	156
15-19	17	1	17
20-24	22	4	88
25-29	27	1	27
Totals:		34	363

So the mean of this collection of grouped data is

$$\bar{x} = \frac{\sum fx}{n} = \frac{363}{34} \approx 10.68.$$

(e) The answers in part (c) and (d) are slightly different because the answer in part (d) is only an approximation to the mean. There are several reasons why it might be skewed, including random fluctuations or some kind of systematic bias (for instance, the data points might cluster near the top of their categories). However, here I believe that the difference between (c) and (d) is acceptable, just random fluctuations.

(f) Arrange the data in increasing order.

$$0, 3, 3, 4, 4, 4, 5, 5, 5, 6, 7, 7, 8, 8,$$
$$8, 10, 10, 11, 11, 11, 11, 11, 11, 12,$$
$$12, 13, 14, 14, 16, 20, 21, 21, 24, 25$$

There are 34 data points. The average of the two middle points is 10.5, so the median is 10.5.

The point with the greatest frequency is 11, so the mode is 11.

$\square$

9.2.8.

x	x^2
15	225
42	1764
53	2809
7	49
9	81
12	144
28	784
47	2209
63	3969
14	196
$\sum x = 290$	$\sum x^2 = 12,230$

The range of the data is $67 - 7 = 56$. The mean of the data is

$$\bar{x} = \frac{\sum x}{n} = \frac{290}{10} = 29.$$

The standard deviation of the data is

$$\begin{aligned} s &= \sqrt{\frac{\sum x^2 - n\bar{x}^2}{n-1}} \\ &= \sqrt{\frac{12,230 - 10(29)^2}{9}} \\ &= \sqrt{424.4} \approx 20.6. \end{aligned}$$

□

9.2.10. Let x represent the midpoint of each interval, and let f represent the frequency of each interval. We have the following table.

Interval	f	x	fx	x^2	fx^2
30-39	1	34.5	34.5	1190.25	1190.25
40-49	6	44.5	267.0	1980.25	11,881.50
50-59	13	54.5	708.5	2970.25	38,613.25
60-69	22	64.5	1419.0	4160.25	91,525.50
70-79	17	74.5	1266.5	5550.25	94,354.25
80-89	13	84.5	1098.5	7140.25	92,823.25
90-99	8	94.5	756.0	8930.25	71,442.00
Totals:	80		5550.0		401,830.00

The mean of this grouped distribution is

$$\bar{x} = \frac{\sum fx}{n} = \frac{5550}{80} = 69.375,$$

and the standard deviation is

$$
\begin{aligned}
s &= \sqrt{\frac{\sum fx^2 - n\bar{x}^2}{n-1}} \\
&= \sqrt{\frac{401,830 - 80(69.375)^2}{79}} \\
&= \sqrt{\frac{16,798.75}{79}} \\
&\approx \sqrt{212.64} \approx 14.6.
\end{aligned}
$$

□

9.2.30.

(a)

$$\bar{x} = \frac{\sum x}{n} = \frac{1627}{11} \approx 148.$$

The mean salary is \$148.000. The salary of governors in Illinois and New Mexico is \$150,000, so Illinois and New Mexico have governors with salaries closest to the mean.

(b)

$$
\begin{aligned}
s &= \sqrt{\frac{\sum x^2 - n\bar{x}^2}{n-1}} \\
&= \sqrt{\frac{244,927 - 11(147.909090)^2}{10}} \\
&=\approx 20.7.
\end{aligned}
$$

The standard deviation is \$20,700.

(c)

$$
\begin{aligned}
\bar{x} + s &= 148 + 20.7 = 168.7, \\
\bar{x} - s &= 148 - 20.7 = 127.3.
\end{aligned}
$$

Five of the 11 salaries fall between these two values, thus

$$\frac{5}{11}(100\%) \approx 45\%$$

of the governors have salaries within 1 standard deviation of the mean.

(d)

$$
\begin{aligned}
\bar{x} + 3s &= 148 + 3(20.7) = 210.1, \\
\bar{x} - 3s &= 148 - 3(20.7) = 85.9.
\end{aligned}
$$

All 11 salaries fall between these two values, so 100% of the governors have salaries within three standard deviations of the mean.

□