# Math 112 Final Exam Dec 7, 2006

**INSTRUCTIONS — READ THIS NOW**

- This test has 10 problems on 2 pages worth a total of 150 points.
- This is a 150 minute test.

**Question 1** Integrate the following

(a) (10 points) $\int \frac{dx}{2+\sqrt{x}}$

(b) (10 points) $\int x^2 \sin(3x) dx$

**Question 2** Consider the plane region $R$ bounded by the curves $y^2 = 2(x-3)$ and $y^2 = x$.

(a) (7 points) Sketch the region $R$, and find its area.

(b) (10 points) Find the volume of the solid generated by rotating $R$ about the $y$-axis.

(c) (10 points) Find the volume of the solid generated by rotating $R$ about the $x$-axis.

**Question 3** Show that the circumference of a circle of radius 1 is $2\pi$ by

(a) (10 points) using $x^2 + y^2 = 1$ as a description of the circle and integrating the appropriate function of $x$ to get arc length;

(b) (10 points) using the parametrization $(\cos t, \sin t), 0 \leq t \leq 2\pi$ as a description of the circle and integrating the appropriate function of $t$ to get arc length.

**Question 4** (8 points) Evaluate the improper integral

$$\int_0^1 \ln(x)\, dx$$

or, if it does not converge, explain why.

**Question 5** (7 points) Compute $\lim_{n\to\infty} \frac{e^{\sin^2(n)}}{n}$.

**Question 6** (10 points) Evaluate

$$\sum_{n=1}^{\infty} \ln\left(\frac{2+\frac{1}{n}}{2+\frac{1}{n+1}}\right).$$

**Question 7** Determine whether the series converges absolutely, converges conditionally, or diverges.

(a) (7 points) $\sum_{n=0}^{\infty} \frac{n^2 e^{-2n}}{n!}$

(b) (7 points) $\sum_{n=0}^{\infty} \frac{(-1)^n (n^2+1)}{n^3+2}$

(c) (7 points) $\sum_{n=0}^{\infty} \frac{(-1)^n (n^3+2)}{n^2+1}$

**Question 8**

(a) (7 points) Find the radius of convergence and interval of convergence for the power series

$$\sum_{n=0}^{\infty} 7^n (x+1)^n.$$

(b) (10 points) Let $f(x)$ be the function defined by the above power series. Find the coefficients $a_n$ in the power series $\sum_{n=0}^{\infty} a_n (x+1)^n$ which represents $xf(x)$.

**Question 9** (10 points) Find the Maclaurin series for the function

$$f(x) = \frac{x}{\sqrt[3]{1-2x}}.$$

What is the radius of convergence of this series? Hint: You may wish to consider the series for $(1+x)^{-1/3}$ first.

**Question 10** (10 points) Find the first five nonzero coefficients in the power series representation $y(x) = \sum_{n=0}^{\infty} a_n x^n$ of the solution of the differential equation

$$(1-x)y''(x) + y(x) = 0$$

with initial conditions $y(0) = 24$, $y'(0) = 12$.