Math 112 Prelim 1
Sep 26, 2006

Name: _____
Instructor: _____
Section: _____

INSTRUCTIONS — READ THIS NOW

• This test has 6 problems on 8 pages worth a total of 100 points. Look over your test package **right now**. If you find any missing pages or problems please ask a proctor for another test booklet.

• Write your name, your instructor's name, and your section number **right now**.

• Show your work. To receive full credit, your answers must be neatly written and logically organized. If you need more space, write on the back side of the preceding sheet, but be sure to label your work clearly.

• This is a closed book exam. No calculators are allowed. You may bring a single crib sheet in your own handwriting (both sides allowed).

• This is a 90 minute test.

OFFICIAL USE ONLY
1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
Total: _____

Problem 1. Evaluate the following integrals.

(a) (7 points) $\int \ln(x + x^2)dx$ (Hint: integration by parts)

(b) (7 points) $\int \frac{2x^2-8x+8}{x^3+x}dx$

(c) (7 points) $\int \frac{dx}{1+x^{1/3}}$

(d) (7 points) $\int_0^1 \frac{e^x}{1+e^x} dx$

Problem 2. (10 points) Compute

$$\frac{d}{dx}\left(x\int_0^{x^2}\sin(t^2)dt\right)$$

Problem 3. Let $I_n = \int_0^\infty x^n e^{-x} dx$.

(a) (5 points) Evaluate I_0.

(b) (10 points) Find a reduction formula for I_n in terms of I_{n-1}.

(c) (5 points) What is I_5?

Problem 4. Let R be the region inside the circle $x^2 + y^2 = 20$ and above the parabola $y = x^2$.

(a) (7 points) Write an integral that gives the area of R.

(b) (7 points) Evaluate the integral (you don't have to simplify your answer - but make sure there are no integral signs in your answer).

Problem 5. Use the fact that $\ln(x) = \int_1^x \frac{1}{t} dt$ to approximate the value of $\ln 4$ by a numerical integration, subdividing the interval into three subintervals (you don't have to simplify your answers),

(a) (6 points) using the midpoint rule

(b) (6 points) using the trapezoidal rule

(c) (6 points) Sketch a region whose area represents the error in the trapezoidal rule approximation, and use this sketch to answer the following question: is the approximation greater than $\ln 4$ or less than $\ln 4$?

Problem 6. (10 points) For which values of $p > 0$ does the integral $\int_1^\infty \frac{x}{1+x^p} dx$ converge? For which values of $p > 0$ does it diverge? Justify your answer by comparison. (Hint: you may use the fact that $\int_1^\infty x^{-p}$ converges if and only if $p > 1$.)