Question 1 Find the volume of the solid formed by revolving the region

$$0 \leq x \leq 1, \quad 0 \leq y \leq 1 + x^2$$

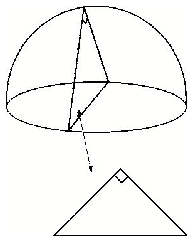
about the y-axis. (Hint: a sketch may help.) In each case, indicate clearly the integral that gives the volume, and then evaluate the integral.

(a) (10 points) by cylindrical shells

(b) (10 points) by disks.

Question 2 (10 points) Consider a solid whose base is a disc of radius r centimeters, and whose cross-section perpendicular to the base is an isosceles right triangle whose hypotenuse rests on the base.

Find the volume of this solid.



Question 3 (15 points) Let ℓ be a positive number. Find b such that the arc length of

$$y = (2/3)(x-1)^{(3/2)}$$

from 0 to b is equal to ℓ.

Question 4 (10 points) Find the surface area of the cone generated by rotating the line segment $y = ax$, $0 \leq x \leq 1$, around the x-axis. Do not count the base of the cone when calculating surface area. (Assume $a \neq 0$, but not necessarily positive.)

Question 5 Consider the parametric curve $x = \sin t, y = t^2$ for $-\pi \leq t \leq 0$.

(a) (5 points) Find a Cartesian equation for the curve by eliminating the parameter.

(b) (5 points) Sketch the curve, indicating the direction with an arrow.

(c) (10 points) Write an integral (in terms of t) that gives the area of the surface obtained by rotating the curve about the y-axis. You do **not** need to simplify or evaluate the integral.

Question 6 Consider the plane curve C traced out by the parametric equations

$$x(t) = e^t + e^{-t}, \qquad y(t) = e^t - e^{-t}, \qquad -\infty < t < \infty.$$

You may use the fact that the curve has no loops or self-intersections.

(a) Compute $x'(t)$ and $y'(t)$, and use these to determine the values of t at which

(i) (5 points) the tangent line is horizontal

(ii) (5 points) the tangent line is vertical

(iii) (5 points) the plane curve fails to be smooth.

(b) (10 points) Find a parametric representation for the tangent line to the curve at the point corresponding to $t = 1$.