

## Instructions:

- 1) Write your name and your instructor's name on the front cover of the exam booklet. (Also put down your section number if you know it.)
- 2) Show all your work. A correct answer may get little credit unless it is clear that you use a correct method or give an adequate reason.
- 3) Do not simplify your answers unless required.
- 4) Do the problems in any order you like, but label the answers clearly.
- 5) This is a closed book test. Calculators, notes, or crib sheets are not allowed.

Some of the following formulas may be useful:

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta) \quad \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta) \quad \tan^2\theta = \sec^2\theta - 1$$

- 1) a)(10 points) Find the derivative of  $G(x)$  if  $G(x) = \int_x^1 e^{t^2} dt$   
b)(5 points) Suppose  $\int_0^2 f(t) dt = 3$ . Calculate  $\int_0^1 f(2t) dt$  (Try a substitution).

2) (30 points) Find the following integrals:

$$\begin{array}{lll} \text{a) } \int x \ln x \, dx & \text{b) } \int e^{\sqrt{x}} \, dx & \text{c) } \int_0^{\pi/2} \sin^2 x \, dx \\ \text{d) } \int \frac{dx}{x^3 - x} & \text{e) } \int_0^{3/2} \frac{x+1}{4x^2+9} \, dx & \end{array}$$

3)(20 points) Let R denote the region bounded by the graphs of  $y = 2 - x^2$  and  $y = -2$ .

- a) Sketch this region.
- b) Write an integral representing the area of R.
- c) Evaluate the integral you found in part (b).

4) (15 points) Determine whether the following integral converges or diverges:

$$\int_0^{\infty} \frac{\cos x}{x^2 + 1} \, dx$$

OVER

5)(20 points)

a) Show that  $\pi/4 = \int_0^1 \frac{dx}{1+x^2}$

b) Use the Midpoint Rule with  $n = 2$  to get an approximate value  $M_2$  for the integral in a). (Do not simplify your answer.)

c) Use the error bound for the Midpoint Rule:  $E_n \leq \frac{K(b-a)^3}{24n^2}$  to show that if  $M_2$  is your answer in part b) then:  $|\pi/4 - M_2| \leq 1/8$ .

Do not try to get the smallest value of  $K$  which works! (Here  $E_n$  is the error in the  $n^{\text{th}}$  Midpoint approximation  $M_n$ ,  $n$  the number of subdivisions,  $[a, b]$  is the interval of integration,  $K$  is an upper bound on  $[a, b]$  of  $|f''(x)|$  where  $f(x)$  is the function being integrated.)