1. Evaluate each of the following integrals:
   a. (8 points) $\int \sin^3 x \cos^4 x \, dx$
   b. (8 points) $\int \sin(\sqrt{x}) \, dx$
   c. (8 points) $\int \frac{3x^2+2}{x^3+x} \, dx$
   d. (8 points) $\int_1^9 \sqrt{(x-1)(9-x)} \, dx$
2. (6 points) A solid is obtained by rotating the region between $y = 4 - x^2$ and the $x-$axis about the line $y = -1$. Find the volume of the solid.
3. (6 points) Write an integral that represents the length of the parabolic curve $y = x^2$, $-1 \le x \le 1$.
4. (6 points) Find the solution to the differential equation $y' = \frac{2x}{y + x^2 y}$ that satisfies $y(0) = -2$.
5. Determine whether each of the following series converges or diverges.
   If they diverge, provide a reason. If they converge, find their sum.
   a. (6 points) $\sum_{n=1}^{\infty} \frac{3^n}{2^{2n}}$
   b. (6 points) $\sum_{n=0}^{\infty} [\tan^{-1}(n) - \tan^{-1}(n+1)]$
6. Determine whether each of the following series converges or diverges. In addition, if the series converges, determine whether the series will also converge absolutely. Justify your answer by quoting and applying the appropriate test.
   a. (6 points) $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$
   b. (6 points) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$
   c. (6 points) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2+1}$
7. a. (5 points) Find the radius and interval of convergence of the power series,
   $$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)\cdot(2n+1)!} = x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} + \cdots$$
   b. (5 points) Let $f(x)$ be the function defined by the series above. Find $f'\left(\frac{\pi}{2}\right)$.
   Hint: First relate the series of $f'(x)$ to another well-known series.
8. (10 points) Solve the differential equation $y'' - xy' - y = 0$ with initial conditions $y(0) = 1$ and $y'(0) = 2$ by using power series. Your answer should be given in the form of a series.

Math 112, Spring 2003
Final Exam
5/12/03

Directions: You may do the problems in any order. Write your solutions in the exam booklet provided, labelling each one clearly. Calculators are not allowed on this exam. There are 150 points total on the exam.

1. (10 points) Approximate $\int_0^4 x^2 dx$ using:
   (a) the Trapezoidal rule with $n = 4$
   (b) Simpson's rule with $n = 4$.
2. (10 points) S is a solid whose base is a disk of radius 3. Cross-sections of S perpendicular to the base are squares. Find the volume of S.
3. (15 points) Find a power series solution to the differential equation
$$y'' = xy' + y.$$
4. (12 points)
   (a) Find the McLaurin series of the function $f(x) = e^{2x}$.

6. (15 points)
   For each of the sequences below, if the sequence converges, find its limit, or if the sequence diverges briefly explain why it diverges.
   (a) $\lim_{n\to\infty} \frac{2n^2}{n^2+3}$
   (b) $\lim_{n\to\infty} \frac{n\cos(n)}{n^2+1}$
   (c) $\lim_{n\to\infty} \frac{1}{\sin(\frac{1}{n})}$
7. (20 points) Say whether each of the following series absolutely converges, converges, or diverges. State what test you are using. Do not try to find the sum of the series.
   (a) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3-1}}{n^2+1}$
   (b) $\sum_{n=1}^{\infty} \frac{\sin n + \cos n}{2^n}$
   (c) $\sum_{n=1}^{\infty} \frac{n^3+2n}{2^n}$
   (d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n^3)}$

9. (20 points) Evaluate the integrals:

(a) $\int_1^2 \frac{x^2+1}{\sqrt{x}}\,dx$

(b) $\int_{-1}^{0} \frac{3t}{(t-1)(t+2)}\,dt$

(c) $\int_0^2 x\sqrt{4-x^2}\,dx$

(d) $\int x^2 \ln x\,dx$

10. (8 points) Find all values of a so that the following integral converges

$$\int_{-\infty}^{0} e^{ax}\,dx.$$

11. (10 ponts) Tell whether each of the following integrals converges or diverges. Justify your answer.

(a) $\int_1^5 \frac{dx}{\sqrt{x-1}}$

(b) $\int_2^{\infty} \frac{x}{\sqrt{x^3-1}}\,dx$

12. (10 points) Find the arc length of the arc given parametrically by the equations

$$x(t) = \int_0^t sin(\theta^2)d\theta$$

$$y(t) = \int_0^t cos(\theta^2)d\theta$$

for $0 \leq t \leq \pi$