Oct. 25, 2007 Math 112, Prelim 2 Time: 7.30 -9p.m.

Instructions:
1) Write your name, your instructor's name, and the section number on the front cover.
2) Show all your work. A correct answer may get little credit unless the method is correct.
3) Do the problems in any order you like, but label the answers clearly.
4) This is a closed book test. Calculators, notes, or crib sheets are not allowed.

The four problems are worth 25 points each. Parts of the same problem have roughly equal weight.

1) Consider the region R enclosed by the curves $y = x^2$ and $y = \sqrt{x}$.

a) Use the slicing method ("washer" shaped slices) to find the volume of the solid obtained by rotating R around the x-axis.

b) Use the method of cylindrical shells to find the volume obtained by rotating R around the y-axis.

2 bonus points if you can explain any obvious relationship between the answers to a) and b).
NOTE: R is rotated around *different* axes in a) and b).

2) Consider the curve $y = \sqrt{x}$ above the x-axis from $x = 0$ to $x = 4$.

(a) Find its arc length by integrating with respect to y. (The following formula may be helpful: $\int \sec^3\theta\, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$)

(b) Find the area of the surface that is obtained by rotating this curve around the x-axis.

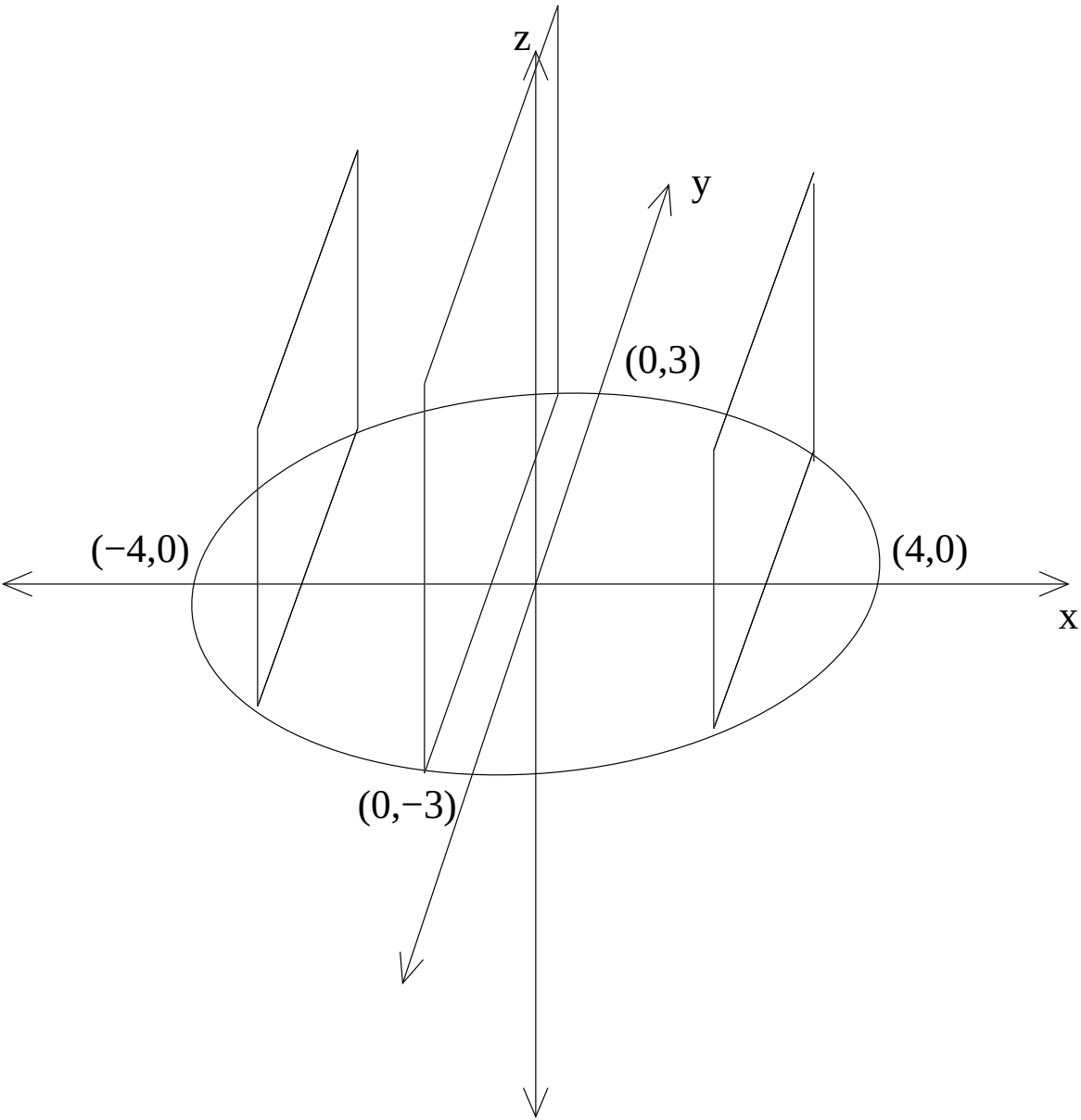
(In both parts of this problem roughly two-thirds of the credit is for setting up the correct definite integral. Ration your time accordingly.)

3) A solid has a flat base whose boundary is the ellipse given by the equation $9x^2 + 16y^2 = 144$. Every cross-section of the solid perpendicular to the x-axis is a square. Find the volume of the solid. (See the figure on the back.)

4) Given the parametric curve, for $0 \le t \le 2\pi$,

$$x = 2\cos t + 6t$$
$$y = 2\sin t$$

a) Find $\frac{dy}{dx}$ (as a function of t)
b) At which point(s) (x, y) on the curve is the tangent line to the curve horizontal?
c) Is the tangent line to the curve ever vertical? Explain.


z
y
(0,3)
(−4,0)
(4,0)
x
(0,−3)