Surfaces review questions

In the following questions, when the surface involved is a quadric surface, identify the type of surface by looking at the horizontal and vertical traces, and sketch it.

Question 1.

Parametrize the following surfaces

(a) $x^2 + 2y^2 + 3z^2 = 1$ for $y \leq 0$,

(b) $4x^2 - 4y^2 - z^2 = 4$ for $0 \leq x \leq 2$,

(c) the torus obtained by rotating the circle in the xz–plane given by $(x-a)^2 + z^2 = R^2$, for $R < a$, about the z–axis.

Hint: think about what your two parameters should represent geometrically.

Question 2.

Let S be the part of $z = x^2 + y^2$ that lies under the plane $z = 4$. Evaluate $\iint_S z \, dS$.

Question 3.

Let S be the same surface as in Question 2, and let $\mathbf{F} = \langle x, xz, xy \rangle$.

(a) Calculate $\text{curl}(\mathbf{F})$ and $\iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$, where we take the orientation on S given by upward pointing normal vectors.

(b) Verify Stokes' Theorem holds.

Question 4.

Let S be the portion of the surface $z^2 = 3x^2 + 3y^2$ between the planes $z = 1$ and $z = 3$. Evaluate $\iint_S x^2 z^2 \, dS$.

Question 5.

Let S be the same surface as in Question 4, oriented with upward pointing normals. Use Stokes' Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \text{curl}(\mathbf{A})$, where $\mathbf{A} = \langle 0, xy, xyz \rangle$.