Math 2210 Homework 4 Solutions

2.1

6. a. $A\mathbf{b}_1 = \begin{bmatrix} 0 \\ -3 \\ 13 \end{bmatrix}$, $A\mathbf{b}_2 = \begin{bmatrix} 14 \\ -9 \\ 4 \end{bmatrix}$, $AB = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}$

   b. $AB = \begin{bmatrix} 4\cdot 1 - 2\cdot 2 & 4\cdot 3 - 2(-1) \\ -3\cdot 1 + 0\cdot 2 & -3\cdot 3 + 0(-1) \\ 3\cdot 1 + 5\cdot 2 & 3\cdot 3 + 5(-1) \end{bmatrix} = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}$

16. a. False. AB must be a 3×3 matrix, but the formula for AB implies that it is 3×1. The plus signs should be just spaces (between columns). This is a common mistake.

   b. True. See the box after Example 6.

   c. False. The left-to-right order of B and C cannot be changed, in general.

   d. False. See Theorem 3(d).

   e. True. This general statement follows from Theorem 3(b).

24. Take any $\mathbf{b}$ in $\mathbb{R}^m$. By hypothesis, $AD\mathbf{b} = I_m\mathbf{b} = \mathbf{b}$. Rewrite this equation as $A(D\mathbf{b}) = \mathbf{b}$. Thus the vector $\mathbf{x} = D\mathbf{b}$ satisfies $A\mathbf{x} = \mathbf{b}$. This proves that the equation $A\mathbf{x} = \mathbf{b}$ has a solution for each $\mathbf{b}$ in $\mathbb{R}^m$. By Theorem 4 in Section 1.4, A has a pivot position in each row. Since each pivot is in a different column, A must have at least as many columns as rows.

2.2

10. a. False. The product matrix is invertible, but the product of inverse should be in the reverse order. See Theorem 6(b).

   b. True, by Theorem 6(a).

   c. True, by Theorem 4.

   d. True, by Theorem 7.

   e. False. The last part of Theorem 7 is misstated here.

32. Not invertible

2.3

8. The 4×4 matrix is invertible, by the Invertible Matrix Theorem, because it is already in echelon form and has four pivot columns.

16. No because the statement (h) of the Invertible Matrix Theorem is then fale. A 5×5 matrix cannot be invertible when its columns do not span $\mathbb{R}^5$.

22. Statement (g) of the Invertible Matrix theorem is false for H, so statement (d) is false, too. That is the equation $H\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

4.1

16. Not a vector space because the zero vector is not in W.

18. $S = \left\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

32. Both H and K contain the zero vector of V because they are subspaces of V. Hence $\mathbf{0}$ is in $H \cap K$. Take $\mathbf{u}$ and $\mathbf{v}$ in $H \cap K$. Then $\mathbf{u}$ and $\mathbf{v}$ are in both H and K. Since H is a subspace, $\mathbf{u}+\mathbf{v}$ is in H. Likewise $\mathbf{u}+\mathbf{v}$ is in K. Hence $\mathbf{u}+\mathbf{v}$ is in $H \cap K$. For any scalar c, the vector $c\mathbf{u}$ is in both H and K because they are subspaces. Hence $c\mathbf{u}$ is in $H \cap K$. Thus $H \cap K$ is a subspace.
The union of two subspaces is not, in general, a subspace. In $\mathbb{R}^2$, let H be the x-axis and K the y-axis. The sum of a nonzero vector in H and a nonzero vector in K is not on either the x-axis or the y-axis. So $H \cup K$ is not closed under vector addition, and $H \cup K$ is not a subspace of $\mathbb{R}^2$.

4.2

4. $\begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

16. $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

28. The two systems have the form $A\mathbf{x} = \mathbf{v}$ and $A\mathbf{x} = 5\mathbf{v}$. Since the fist system is consistent, $\mathbf{v}$ is in Col A. Since Col A is a subspace of $\mathbb{R}^3$, $5\mathbf{v}$ is also in Col A. Thus the second system is consistent.

34. The kernel of T is $\{\mathbf{0}\}$.