MATH 3320, HOMEWORK #10

DUE FRIDAY, NOVEMBER 2

To ensure that you get full credit, be sure to *show your work* in the problems that require calculations. Very little credit is given for answers without justification. Please write in complete sentences to help us understand what you are doing.

You may collaborate with classmates in solving the problems, including the extra credit problems. If you do so, please list their names on your assignment. However, you should not consult *any* other people (except the instructors or TAs), or use online resources. (Seriously, it's very obvious to us when this occurs, and there are drastic consequences, so don't do it!) If you use results that were not proved in class, please provide your own proof.

1.

(a) (5 points) Find the simple continued fraction expansion of $\frac{771}{356}$.
(b) (10 points) Use your answer in (a) to find an integer solution of $771x + 356y = 1$.

2.

(a) (10 points) Write the continued fraction $\alpha = [1, 2, 3, 4, 5, 6]$ as a rational number $\frac{p}{q}$ where p and q are coprime.
(b) (10 points) Consider the periodic continued fraction $\alpha = [3, 1, \overline{5, 1}]$. Find a quadratic polynomial $p(x) = ax^2 + bx + c$ with $a, b, c \in \mathbf{Z}$ such that $p(\alpha) = 0$. Which root of the polynomial is α?

3.

(a) (10 points) Find the periodic simple continued fraction expansion for $\sqrt{7}$.
(b) (10 points) Use your answer in (a) to find a rational number r such that $\left|r - \sqrt{7}\right| < 10^{-6}$.

4. (15 points) Prove that there are infinitely many integers $n \in \mathbf{N}$ such that the sum of the first n integers is a perfect square.

5. (Extra Credit) Suppose that you see a long row of blocks, numbered 1, 2, 3, 4, etc. from left to right. You have been told that there are somewhere between 50 and 500 blocks in the row. There is a certain block B such that the sum of all the numbers on the blocks to the left of B is equal to the sum of all the numbers on the blocks to the right of B.

(a) (5 points) What number is on block B? How many blocks are there total?
(b) (5 points) What are all the possible solutions to the problem if you allow for an arbitrary number of blocks in the row? (Hint: Continued fractions)