# MATH 3320, HOMEWORK #11

DUE FRIDAY, NOVEMBER 9

To ensure that you get full credit, be sure to *show your work* in the problems that require calculations. Very little credit is given for answers without justification. Please write in complete sentences to help us understand what you are doing.

You may collaborate with classmates in solving the problems, including the extra credit problems. If you do so, please list their names on your assignment. However, you should not consult *any* other people (except the instructors or TAs), or use online resources. (Seriously, it's very obvious to us when this occurs, and there are drastic consequences, so don't do it!) If you use results that were not proved in class, please provide your own proof.

*A word on notation.* Davenport writes continued fractions using square brackets, but angled brackets are also common. In this homework, we'll treat them as the same, that is,

$$\alpha = [a_0, a_1, a_2, \ldots] = \langle a_0, a_1, a_2, \ldots \rangle .$$

(Note that Davenport uses $[a_0, a_1, \ldots]$'s to stand for just the *numerator* of the expansion. For this homework, we will *not* follow this convention.)

**1.**

(a) (10 points) Let $\frac{A}{B} > 1$ be a rational number with $\gcd(A, B) = 1$ and $A, B > 0$. If $[a_0, \ldots, a_n]$ is the simple continued fraction expansion of $\frac{A}{B}$, what is the simple continued fraction expansion of $\frac{B}{A}$?

(b) (10 points) Let $\frac{p_k}{q_k}$ be the convergents of a real number $\alpha = [a_0, \ldots, a_n, \ldots]$. Show that $q_k \geq 2^{k/2}$ if $k \geq 2$. (*Hint:* Use induction. Recall that the recursion formulas are $p_{-1} = 1$, $q_{-1} = 0$, $p_0 = a_0$, $q_0 = 1$, and $p_{n+1} = a_{n+1}p_n + p_{n-1}$, $q_{n+1} = a_{n+1}q_n + q_{n-1}$.)

**2.**

(a) (10 points) Let $p_k/q_k$ be the $k$-th convergent of $\alpha = [a_0, \ldots, a_n]$. If $a_0 \geq 1$, prove that $p_k/p_{k-1} = [a_k, \ldots, a_0]$ and $q_k/q_{k-1} = [a_k, \ldots, a_1]$.

(b) (10 points) A certain number $\alpha$ has a continued fraction expansion whose first three partial quotients are $a_0 = 1$, $a_1 = 3$, $a_2 = 4, a_3 = 6$ and assume that $\alpha$ has at least one more partial quotient $a_4$. What can you say about an upper and a lower bound for $\alpha$?

**3.**

(a) (10 points) Find the best approximation to $\sqrt{2}$ among all rational numbers with denominator not exceeding 985. (Hint: Use the fact that the $k$th convergent $p_k/q_k$ is the best rational approximation for all rational numbers with denominators $\leq q_k$.)

(b) (10 points) Without computing the continued fraction expansion of $\alpha = \frac{1+\sqrt{3}}{2}$, decide if $153/112$ and $571/418$ are consecutive convergents of $\alpha$.

**4.** (15 points) Show that $\sqrt{9n^2 + 6} = [3n, \overline{n, 6n}]$.

**5.** (Extra Credit) A baseball player's *batting average* is defined to be the number of hits divided by the number of at-bats and is usually rounded to the nearest thousandth (e.g. .1235 rounds up .124;

.2444 rounds to .244, etc.). For example, this year's MLB batting average leader Mookie Betts had a .346 batting average.

For (10 points), if a player has a batting average of .346, what is the smallest possible number of times that this player has been at bat?