Math 3320 Prelim   Name: ______________________

November 21st, 2017

INSTRUCTIONS

• Print your name **right now**.

• This test has **7 problems**. Please carefully write all your final answers on the page where they are posed.

• Look over this test **as soon as the prelim begins**. If you find any missing pages or problems please ask a proctor for another copy.

• *Show your work.* To receive full credit, your answers must be neatly written, and logically organized. If you need more space, write on the back side of the preceding sheet, but be sure to label your work clearly.

• Scrap paper is available for rough work. You may not hand in work on scrap paper.

• You have 90 minutes to complete this prelim.

• This is a closed book exam and no notes are allowed. You are *not* allowed to use a calculator, cell phone, or any other electronic devices.

• Academic integrity is expected of all Cornell University students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Please sign below to indicate that you have read and agree to these instructions.

____

Signature of Student

OFFICIAL USE ONLY
(do not fill in)

1. ______/10
2. ______/10
3. ______/10
4. ______/10
5. ______/10
6. ______/10
7. ______/10

Total: ______/70

1.) Prove that 101 divides $10^{14502}+1$.

2.) Find all the solutions to the equation

$$x^{21} \equiv 3 \pmod{61}.$$

Use that 2 is a primitive root modulo 61 and express your solutions as powers of 2.

3.) Solve

$$37x \equiv 3 \pmod{50}.$$

4.) Does $x^2 \equiv 151 \pmod{211}$ have a solution? [You may use that 151 and 211 are primes.]

5.) A band of fifteen pirates upon dividing their gold coins evenly amongst themselves found that three were left over. In the ensuing brawl, eight pirates was killed. The entire hoard was again redistributed equally among the remaining pirates, but now two coins remained. Another altercation broke out and three additional pirates were killed. They tried it again, and this time the coins were distributed evenly with none left over. What is the least number of coins they could have started with?

6.) Let m and n be positive integers with $\gcd(m, n) = 1$. Prove that

$$m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}.$$

7.) Fix a prime $p \geq 11$. Prove that there is an integer $1 \leq n \leq 9$ such that n and $n+1$ are both squares modulo p.
[Hint: 1, 4 and 9 are always squares modulo p]