MATH 321 Manifolds and Differential Forms (II)

Homework 6 Solution

Due October 18, 3:00 p.m.

4.12 (6 points)

(i) Proof: $(dx_I, dx_J) = \delta_{I,J}$ is obvious, so is the fact that dx_I's form an orthonormal basis of the space of constant k-forms. $\square$

(ii) Proof: Let $\alpha = \sum_I a_I dx_I$, then $(\alpha, \alpha) = \sum a_I^2 \geq 0$ and $(\alpha, \alpha) = 0$ iff $\alpha = 0$. $\square$

(iii) Proof: $\alpha(*\beta) = \alpha \sum_I b_I \varepsilon_I dx_{I^c} = \sum_I a_I dx_I \sum b_I \varepsilon_I dx_{I^c} = \sum_I a_I b_I \varepsilon_I dx_I dx_{I^c} = \sum_I a_I b_I dx = (\alpha, \beta) dx_1 dx_2 \ldots dx_n$. $\square$

(iv) Proof: By (iii), $\beta(*\alpha) = (\beta, \alpha)dx = (\alpha, \beta)dx = \alpha(*\beta)$. $\square$

(v) Proof: $(*\alpha, *\beta)dx = *\alpha(**\beta) = (*\alpha)\beta = \alpha(*\beta) = (\alpha, \beta)dx$.So $(*\alpha, *\beta) = (\alpha, \beta)$. $\square$

4.13 (6 points) Solution: Just tedious computations. (Omitted). $\square$

5.6 (5 points)

(i) Solution: $\sigma^{-1} = (341652)$, $\tau^{-1} = (625314)$, $\sigma\tau = (562413)$ and $\tau\sigma = (415236)$. $\square$

(ii) Solution: $\sigma^{-1} = (21345 \ldots n)$, $\tau^{-1} = (n23 \ldots (n-2)(n-1))$, $\sigma\tau = (n1345 \ldots (n-2)(n-1)2)$ and $\tau\sigma = (2n345 \ldots (n-2)(n-1))$. $\square$

5.7 (3 points) Proof: Straightforward. $\square$