Math 418

FINAL EXAMINATION (Take home)

Due May 14, 2001; 3 p.m.

You may use any books and notes but do not discuss your work with anybody except me. If you have any questions or comments to the problems, you may communicate with me by e-mail (ebd1@cornell.edu) or by phone (273-1071). You can discuss your work with me by e-mail or by phone and you can see me on Saturday, May 12 at 2:30 p.m. in my office (Malott 535). Exam papers may be placed into my mailbox or under the door of Malott 535.

1. (50 points; 10 points for every part)

(a) Classify the singular points on the extended plane for the function $f(z) = (1+z^4)^{-1}$.

(b) Evaluate the residue at every isolated singular point.

(c) Compute, for $R > 1$, the integral $\int_{C_R} f(z)dz$ where C_R is a contour which consists of a circular arc (R, iR) and two line segments [iR,0] and $[0, R]$.

(d) Passing to the limit as $R \to \infty$, evaluate $\int_0^\infty (1+x^4)^{-1}dx$.

(e) Find the radius of convergence of the Taylor series for $f(z)$ about the point $t(1+i)$ where $t > 0$.

2. (25 points) Let P be a polynomial of degree n and let $C_r = \{|z| = r\}$ counterclockwise oriented. Find

$$\lim_{r\to\infty} \int_{C_r} \frac{P'(z)}{P(z)} dz.$$

3. (25 points) Find all values of real parameters s and t for which

$$u = x^2 - sy^2,$$
$$v = txy$$

is an analytic mapping from the xy-plane to the uv-plane.

4. (50 points; 5 points for each part). Which of the following statements are true and which are false? Explain your answers.

(a) If α is a pole for f and for g, then it is a pole for fg.

(b) If $u(z)$ is a harmonic function in $\{|z| < 1\}$, then so is $v(z) = u(z^2)$.

(c)$|\sin\ z| \leq |z|$ for all complex z.

(d) If P is a polynomial, then $\int_C P(z)^{-2}dz = 0$ for every Jordan contour C which does not pass through zeros of P.

(e) If α is a simple pole for f, then it is a pole of order 2 for f'.

(f) If α is a pole of order 2 for f', then it is a simple pole for f.

$(g) w = z^2$ is a conformal mapping of the unit disk $\{|z| < 1\}$ onto itself.

(h) The series $1 + z + ... + z^n + ...$ converges uniformly in the unit disk $\{|z| < 1\}$.

(i) If f is entire function and if $f(z) \geq 0$ for all $z \in \mathbf{C}$, then f is a constant.

(j) Let $w = f(z)$ be a 1-1 mapping from a domain D onto a domain D'. If f is analytic, then the inverse mapping is analytic as well.