Math 418

COMPLEX VARIABLES)

Instructor E..B.Dynkin

Home page: http://www.math.cornell.edu/ ebd/

Class time: Tuesday, Thursday 11:40-12:55, Malott Hall, Room 224
Office hours: Tuesday & Thursday, 9:30-10:10 a.m. or by appointment
T.A.:Yan Zeng; yanzeng math. cornell.edu; phones 255-7554(office), 257-7140(home)
office hours on Monday, 2:00-3:00, Wednesday, 2:00-3:00, Mallott Hall, Room 218 and by appointment

Textbook: N. Levinson and R. M. Redheffer, Complex Variables.

Homework: Problem sets will be graded by the T. A. Written solutions for a part of the problems will be placed on the WEB site (www.math.cornell.edu/ yanzeng).

Reading: It is recommended to read each chapter of the textbook twice: before and after the discussion in class.

The grades will be based on the score: 25% for homework, 25% for prelim. 50% for final exam.

Homework 1

Due January 30, 2001

C1. Express $\cos n\theta$ and $\sin n\theta$ through $\cos\theta$ and $\sin\theta$ by using the binomial formula and the equation

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta.$$

C2. Let $w = \frac{z-1}{z+1}$. Prove that w is imaginary if and only if $|z| = 1$ Deduce from here: a point z lies on the unit circle centered at 0 if and only if the angle $-1, z, 1$ is right.

C3. Prove that

$$(1+i)^n = 2^{n/2}(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}).$$

By using this formula, compute

$$1 - \binom{n}{2} + \binom{n}{4} - \binom{n}{6} + \ldots$$

and

$$\binom{n}{1} - \binom{n}{3} + \binom{n}{5} - \binom{N}{7} + \ldots.$$

C4. Prove that

$$\alpha\bar{\alpha} = |\alpha|^2, \frac{1}{\alpha} = \frac{\bar{\alpha}}{|\alpha|^2}, \Re\alpha = \frac{\alpha+\bar{\alpha}}{2}, \Im\alpha = \frac{\alpha-\bar{\alpha}}{2i},$$
$$|\alpha+\beta|^2 + |\alpha-\beta|^2 = 2|\alpha|^2 + 2|\beta|^2.$$

C5. What sets on the plane are defined by the conditions:

$$\Re z > 0, b_1 < \Im z < b_2, \Re\alpha z = a, |z-\alpha| = r, |z-\alpha| < r, r_1 \leq |z| \leq r_2, \Re\frac{1}{z} = 1?$$

Here a, b_1, b_2, r, r_1, r_2 are real number and $r > 0, r_2 > r_1 > 0$; α is a complex number.

C6. Express in the form $a + ib$ the numbers

$$(1+i)^2, \frac{3+4i}{1-2i}$$

and the numbers

$$z^3, \bar{z}z, \frac{\bar{z}}{z}, \frac{z-i}{1-i\bar{z}}$$

for $z = x + iy, z \neq 0, z \neq i$.

C7. Prove that $\Re(1/z) > 0$ if and only if $\Re z > 0$.

C8. Describe in the geometrical terms the following transformations:

$$z' = iz, z' = 2z, z' = -z, z' = -2iz.$$

C9. By using that $13 = 2^2 + 3^2$ and $74 = 5^2 + 7^2$, express $962 = 13 \times 74$ as the sum of two squares.

Hint. Use that $|\alpha\beta|^2 = |\alpha|^2|\beta|^2$.