Math 418

Homework 7

Due March 15

C13. Suppose that the series

$$\sum_{-\infty}^{+\infty} a_n z^n \tag{0.1}$$

converges absolutely at points β and γ. Prove that the series above converges uniformly on the circle $|z| = \rho$ for every $|\beta| < \rho < |\gamma|$.

Hint. Use the Weierstrass' test: a series $\sum \varphi_n(z)$ converges uniformly on a set B if $|\varphi_n(z)| \le c_n$ for all $z \in B$ and $\sum c_n < \infty$.

C.14. Let g be an entire function (that is, analytic in the entire plane $\mathbb{C}$). Prove that, if α is an isolated singularity for f, then it is also an isolated singularity for $h(z) = g[f(z)]$. Is it a removable singularity for h if it is a removable singularity for f?

C15!!! Suppose f and g are entire functions and $|f(z)| \le |g(z)|$ for all z. Prove or disprove that $f = cg$ where c is a constant.

C16!!! Suppose that f is analytic at α and $|f'(\alpha)| < 1$. Let

$$z_{n+1} = f(z_n) \quad \text{for } n = 0, 1, 2, \ldots.$$

Clearly, all z_n are determined by a choice of z_0. Prove that there exists $\varepsilon > 0$ such that $z_n \to \alpha$ if $|z_0 - \alpha| < \varepsilon$, provided $f(\alpha) = \alpha$.

Problems in Levinson and Redheffer:
2,3,6 in Chapter 4, Section 2;
1,2a,3 in Chapter 4, Section 3.

Read Chapter 3, Section 9; Chapter 4, Sections 2 and 3.