MATH 418 COMPLEX VARIABLES
Homework 10 Solution

Due April 19, 2001

Note: If you have any questions about the solution, or you think there are some typos/errors in the solution, please e-mail me. I'll double-check it and then reply to you. Thank you.

C19. Solution:
(a) Yes. Since non-degenerate bilinear transformations are 1-1 and onto $\bar{\mathbb{C}}$.
(b) No. $\sin 0 = \sin \pi$.
(c) Yes. $z^2 = z\prime^2$ implies $(z - z')(z + z') = 0$. But $\{|z - 1| < 1\}$ has no points symmetric with respect to 0, so $z + z'$ can't be 0. Hence $z = z'$. □

5. Proof: We only need to show $\left|\frac{h(z)-h(0)}{h(0)}\right| < 1$ for z small enough. Indeed, for z small enough, $\left|\frac{h(z)-h(0)}{z}\right| < 1 + |h'(0)|$ by the definition of derivative. So $\left|\frac{h(z)-h(0)}{h(0)}\right| < (1 + |h'(0)|)\frac{|z|}{|h(0)|} \longrightarrow 0$ as $z \longrightarrow 0$. So, for z small enough, $L(z)$ is well-defined and is an analytic logarithm of $h(z)/h(0)$ near 0. □

6. Proof: $H(z)$ is the compostion of $[h(0)]^{1/n}e^z$ and $L(z)/n$, which is analytic around 0. So, $H(z)$ is analytic for small z and routine computation yields $[H(z)]^n = h(z)$. □

1. Proof: We use the following theorem, which can be found in almost all textbooks: If f and g are analytic in a domain D, $E \subset D$ has a limit point which belongs to D, and $f = g$ on E, then $f = g$ on D.

In this problem, $D = \mathbb{C}$, E is a segment of the real axis. So, these two entire functions agree on $\mathbb{C}$. □

2. Proof: By Theorem 7.6, page 151 in the textbook, $|1/f|$ assumes its maximum value on the boundary. So $|f|$ assumes its minimum value on the boundary. □