MATH 418 COMPLEX VARIABLES
Homework 9 Solution

Due April 10, 2001

Note: If you have any questions about the solution, or you think there are some typos/errors in the solution, please e-mail me. I'll double-check it and then reply to you. Thank you.

C17. Solution: $\triangle P = 0$ implies $a = -c$. Assume $f = P + iQ$ is an analytic function, then $f' = P_x + iQ_x = P_x - iP_y = 2ax + by - i(bx - 2ay) = 2a(x + iy) - ib(x + iy) = 2az - ibz$. So $f(z) = az^2 - ibz/2 + K$ where K is some complex constant. To make $Ref = P$, we should set K as a purely imaginary number. □

C18. Solution: Let $f(z) = z$, $h = Ref$. Then, Poisson formula

$$u(z) = \frac{1}{2\pi}\int_{|\zeta|=R} \frac{R^2 - |z|^2}{|\zeta - z|^2} u(\zeta) d\theta$$

implies

$$\int_0^{2\pi} \frac{1 - |z|^2}{|z - e^{i\theta}|^2} \cos\theta d\theta = \int_{|\zeta|=1} \frac{1 - |z|^2}{|\zeta - z|^2} h(\zeta) d\theta = 2\pi h(z) = 2\pi x$$

□

5.
(a) $2z/(z+1)$
(b) $\frac{z-1}{z+1} i$
(c) z
(d) iz
(e) $\frac{z+i}{z-1}$ □

7. Solution: ω maps $(-1, 0, 1)$ to three distinct points on the unit circle. So ω maps real axis to unit circle. Note $\omega(i) = 0$, so ω maps the upper half plane onto the unit circle. □