Math 1110: **Limits** Aug 27 (Mon)

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Consider the following functions:

$$f(x) = \frac{x^2 + x - 2}{x - 1}, \qquad g(x) = \frac{\sqrt{x^2 + 4} - 2}{x^2}, \qquad h(x) = \frac{\sin x}{x}.$$

1. For each function, determine its domain and list the point at which it is *not* defined.

2. For each function, what kind of expression do you get if you nevertheless try to plug in the point at which they are undefined? (but note that this is "illegal"!).

3. Even if a function $f(x)$ is undefined at a point $x = a$, we can compute the value of the function for points that are very close to a.

   For each function above, compute the value of the function for a few points that are very close to the point where it is undefined. In each case, what do you notice? Do you see a "pattern"?

4. For the function $f(x)$ there is no point a such that $f(a) = 3$ (in other words, $f(x)$ never gives the value 3). But then, one important question is the following: even though $f(x)$ never gives the value 3, can the function get as close as we want to 3? Or put differently, can we pre-determine a level of precision for which the function will "attain" 3?

   Let's say we want $f(x)$ to "attain" the value 3 with a level of precision of 0.1. Find several values of x for which $2.9 < f(x) < 3.1$ (i.e. for which $f(x)$ is between 2.9 and 3.1).

5. Can you find an interval around $x = 1$ for which you can assure that $f(x)$ is going to be between 2.9 and 3.1?

6. Let us now do the same with a level of precision of 0.01, in other words, we now want $f(x)$ to be between 2.99 and 3.01. Can you find an interval around $x = 1$ for which you can assure that $f(x)$ is going to be between 2.99 and 3.01? Does the interval you found at the previous point also work here?

7. Let us finally inquire the same question with the function $h(x) = \frac{\sin x}{x}$. By computing the value of $h(x)$ for points that are close to $x = 0$, we see that $h(x)$ goes to 1. Let's say we want a precision of 0.1, i.e. we want $h(x)$ to be between 0.9 and 1.1.
   Try to find an interval around $x = 0$ for which $h(x)$ is in that range. Does the interval you found for $f(x)$ for a precision of 0.1 work here? Would a bigger interval also work?