Introduction to Probability

Probability is the study of random events, some examples of random events are:

(1) How many people take the subway in NYC on a given day.

(2) How many people are in a city at any particular moment.

(3) The behavior of the stock market.

(4) The population of a species of animals at any moment in time.

(5) The winning lottery number on any given day.

Since we don't know what the outcome of an uncertain event will be, we use probability to study the *expected* outcome. In order to study the expected outcome we have to first know what all the possible outcomes will be.

Definition: The *sample space* of an experiment is all the possible outmcomes of that experiment. We donote the sample space of and experiment by the set S. For example, the experiment consisting of tossing a coin twice is

$$S = \{(H,H),(H,T),(T,H),(T,T)\}.$$

Note that the pair (H,T) represents the outcome where the first time that you toss the coin you get heads and the second time that you toss the coin you get T. *It is not the same as the outcome* (T,H) *!!!*

Exercise Write the sample space for the following experiments.

(1) **Experiment:** Tossing one coin.

(2) **Experiment:** Tossing two coins.

(3) **Experiment:** Picking out a marble from an urn with 2 red marble and 10 black marbles.

(4) **Experiment:** Picking out 3 marbles at once from an urn with 2 red marbles and 10 black marbles.

(5) **Experiment:** Picking out a marble, noting its color, and putting it back into an urn with 2 red marbles, and 10 black marbles three times in row.

Note that a sample space does not necessarily mean that every element of the sample space has equal probability.

Exercise: Using the sample spaces that you calculate in the previous exercise, determine whether these sample spaces have equally likely outcomes.

(1)

(2)

(3)

(4)

(5)

Definition: An *event* is set of possible outcomes from an experiment. So we can denote and event by a set E and say

$$E \subseteq S.$$

Exercise: Write down the sets for the following events.

(1) The event that you see a heads when you toss a coin once.

(2) The event that you see no tails when you toss a coin twice.

(3) The event that you do not see a red marble when you pick 3 marbles at once from an urn with 2 red marbles and 10 black marbles.

Since events are sets we can use set operations to help us understand the intersection, union, and complements of events.
Example: Consider the experiment where we toss a six sided dice. First we write down the state space for this experiment

$$S = \{1, 2, 3, 4, 5, 6\}.$$

(1) Let E be the event that you see an even number. Write down E.

(2) Let F be the even that you see a number that is a multiple of 4. Write down F.

(3) What is the even that you see a number that is both in F and E. Describe it in words.

Definition: Two events E and F are said to be *mutually exclusive* if

$$E \cap F = \emptyset.$$

Example: Let F be the event that when you toss two coins you get a head. Let E be the even that when you toss two coins you get (H, H). What is $E \cap F$?

Now that we know how to analyze all possible outcomes we can estimate the probability of an event.

Definition: Let S be a sample space with equally likely outcomes, and let E be an event contained in S. The probability the E occurs is

$$P(E) = \frac{n(E)}{n(S)}.$$

YOU CAN ONLY USE THIS WHEN THE SAMPLE SPACE HAS EQUALLY LIKELY OUTCOMES!

Exercise: Assume that you have a well shuffled deck of 52 cards. Calculate the following probabilities.

(1) P(Draw a black card)

(2) P(Draw the Queen of Hearts)

(3) P(Draw a 5 or an Ace)

Exercise: Assume that you toss two fair coins. Calculate the following probabilities.

(1) P(You get at least one Head)

(2) P(You get no Tails)

(3) P(You get exactly one Head)