Probability Basics

Recall that if we have a sample space S which has equally likely outcomes then the probability that an event E occurs is

$$P(E) = \frac{n(E)}{n(S)}.$$

It follows that for any two events E and F contained in S we have that

$$P(E \cup F) = \frac{n(E \cup F)}{n(S)}.$$

In the previous chapters we learned that for any two sets E and F the *Union Rule for Sets* (chapter 7.2 of your book) tells us that

$$n(E \cup F) = n(E) + n(F) - n(E \cap F).$$

Question (The Union Rule for Sets): Let S be an equally likely sample space, and let E and F be any two events ocurring in S. What is the probability of $E \cup F$?

Question (The Union Rule for Mutually Exclusive Sets): What is the probability of $E \cap F$ if E and F are mutually exclusive sets?

The Union Rule for Sets applies to any events E and F, eventhough above we assume that they are part of a equally likely sample space.

Exercises:

(1) You roll two fair dice at the same time. What is the probability that you see a two or you see two dice that sum to 6.

(2) You roll a fair dice twice. What is the probability that you see a two the first time you roll the dice or that the sum of both rolls adds up to 6.

(3) A person's eye color is determined by a pair of genes which may contain the gene B (the dominant gene) or b (the recesive gene). If a person has BB Bb of bB then they will have brown eyes, if they carry bb then they will have blue eyes. A mother and a father each give a gene to their child with equal probability. If a mother is BB and the father is bB the possible gene sequences that their child can have are

	B	B
B	BB	BB
b	bB	bB

What is the probability that if a mother is BB and a father is bB that their child will have blue eyes?

If a mother is Bb and a father is bB, what is the probability that their child will have blue eyes or carry the recessive b gene?

Let S be any sample space, it is clear that

$$P(S) = 1.$$

Using this equality we can derive the complement rule that relates the probability of a set to that of its complement.

Question (The Compliment Rule): Let E be an event, and E' be the complement of E? How does $P(E)$ relate to $P(E')$.
(*Hint: Use the fact that* $E \cup E' = S$.)

Exercises: Use the Compliment Rule to solve the following problems.

(1) Give a regular playing deck with 52 cards, what is the probability that when you draw a card at random you do not get a queen.

(2) If you roll a dice, what is the probability that you do not get a prime number?

(3) In 2004, the statisticians P. Diaconis, S. Holmes, and R. Montgomery at Stanford University discovered that if you toss a coin the probability of comming up as started is 0.51 . If you start with a coing heads up, what is the probability that it comes out tails?

Exercise (Chapter 7.4 Problem 59): Color blindness occurs more comonly in males than in females. Let M represent males and C represent the people that are color blind. The following probabilities have been recorded:

$$P(C) = .039, \quad P(M \cap C) = .035, \quad P(M \cup C) = .491$$

(1) Draw a Venn diagram that represents the probabilites for all possible combinations characteristics.

(2) Using the Venn diagram above give the probability for each of the following possible events.

(a) Female and colorblind
(b) Male and not colorblind
(c) Female and not colorblind