Binomial Probabilities

Suppose that an experiment has two outcomes; success or failure. Assume that a success happens with probability p and a failure happens with probability $1 - p$.

Examples of such experiments can be the following:

(1) **Experiment:** Toss a fair coin.
Success: The coin comes out heads.

$P(\text{Success}) =$

$P(\text{Failure}) =$

(2) **Experiment:** Throw a six sided die.
Success: The die lands on either 1 or 2.

$P(\text{Success}) =$

$P(\text{Failure}) =$

(3) **Experiment:** Draw a ball at random from an urn with 2 red balls, 1 white ball 3 blue balls.
Success: Drawing a white ball.

$P(\text{Success}) =$

$P(\text{Failure}) =$

The whole point of this section:

We want to be able to understand what is the probability of obtaining (at least) x successes if we repeat an experiment n times. For example, we want to understand how to calculate the following probabilities.

(1) What is the probability that I get at least 3 heads if I toss a fair coin 5 times?

(2) What is the probability that if I throw a die three times, two out of those three throws will be less than or equal to 2?

(3) Assume that I sample 7 times with replacement from an urn with 2 red ball, 1 white ball and 3 blue balls. What is the probability that I drew the white ball exactly 5 times?

Note that all the experiments above have the following three things in common.

(1) A same experiment is repeated several times.

(2) We only consider two outcomes; we denote these two outcomes as *success* and *failure*.

(3) Each trial of an experiment is independent from the other trials.

Definition: An experiment with characteristics 1-3 listed above is called a *Binomial Experiment.*

Let's do some examples to see how to calculate these probabilities.

Example: I toss a fair coin 5 times. What is the probability that I get 3 heads among the 5 tosses?

(a) How many ways can I get 3 heads if I toss a coin 5 times?

(b) Below is a list of all the ways that I can get 3 heads if I toss a coin 5 times.

HHHTT	THHHT	HHTHT	THHTH	HHTTH
THTHH	HTHHT	TTHHH	HTHTH	HTTHH

What is the probability that any of the sequences above occur?

(c) Use parts (a) and (b) to get the answer to the original question.

Example: I toss a fair coin 5 times. What is the probability that I get at least 3 heads among the 5 tosses?

(a) What is the probability of getting *exactly* 3 heads among the 5 tosses?

(b) What is the probability of getting *exactly* 4 heads among the 5 tosses?

(c) What is the probability of getting *exactly* 5 heads among the 5 tosses?

(d) Use parts (a), (b), and (c) to get the answer to the original question.

Example: Assume that I have a weighted coin so that the probability that it comes out heads is $\frac{1}{3}$. If I toss this weighted coin 5 times, what is the probability that I get 3 heads?

(a) How many ways can I get 3 heads if I toss the coin 5 times?

(b) Below is a list of all the ways that I can get 3 heads if I toss a coin 5 times.

HHHTT	THHHT	HHTHT	THHTH	HHTTH
THTHH	HTHHT	TTHHH	HTHTH	HTTHH

What is the probability that any of the sequences above occur?

(c) Use parts (a) and (b) to get the answer to the original question.

Example: I toss the weighted coin from the last example 5 times. What is the probability that I get at least 3 heads among the 5 tosses?

(a) What is the probability of getting *exactly* 3 heads among the 5 tosses?

(b) What is the probability of getting *exactly* 4 heads among the 5 tosses?

(c) What is the probability of getting *exactly* 5 heads among the 5 tosses?

(d) Use parts (a), (b), and (c) to get the answer to the original question.

In General: Assume that an experiment has only two outcomes; success and failure. The probability that the experiment results in a success is p, which implies that the probability of a failure is $1 - p$. If I run the experiment n times, what is the probability that I get x successes (assuming that $x \leq n$, of course) and if all n trials of the experiment are independent of each other.

(a) How many ways can I get x successes among the n trials?

(b) Assume that a particular list of n trials of the experiment has x successes and $n - x$ failures. What is the probability of this list of n trials?

(c) Use parts (a) and (b) to answer the original question.

Binomial Probability (p. 426): If p is the probability of success in a single trial of a binomial experiment, the probability of x successes and $n - x$ failures in n independent repeated trials of the experiment is

$$\binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$$

Questions:

(a) I toss a dice 6 times. What is the probability that 4 out of the 6 tosses will be less than or equal to 2?

(b) If I sample 5 times with replacement from an urn with 2 red balls, 1 white ball, and 3 blue balls, what is the probability that I draw the white ball at least 4 times?

Question 33-35 (p. 431): *Personnel Screening* A company gives prospective workers a 6-question, multiple choice test. Each question has 5 possible answers, that there is a $\frac{1}{5}$ chain of answering a question correctly by just guessing. Find the probabilities of getting the following results by chance.

33. Exactly 2 correct answers

34. No correct answers.

35. At least 4 correct answers.

Questions 43-46 (p. 431) *Drug Effectiveness* A new drug cures 70% of the people taking it. Suppose 20 people take the drug; find the probabilities of the following.

43. Exactly 18 people are cured.

44. Exactly 17 people are cured.

46. At least 18 people are cured.

***** At most 19 people are cured.

Question 56 (p. 432) *Flu Inoculations* A flu vaccine has a probability of 80% of preventing a person who is inoculated from getting the flu. A count health office inoculates 83 people. Find the probabilities of the following.

(a) Exactly 10 of the people inoculated get the flu.

(b) No more than 4 people inoculated get the flu.

(c) None of the people inoculated get the flu.